An Analogue of the Liouville Theorem for Linear Relations in Banach Spaces

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Consider a bounded linear operator T between Banach spaces $\mathcal{B}$, $\mathcal{B}'$ which can be decomposed into direct sums $\mathcal{B} = \mathcal{B}_1 \oplus \mathcal{B}_2$, $\mathcal{B}' = \mathcal{B}'_1 \oplus \mathcal{B}'_2$. Such linear operator can be represented by a 2×2 operator matrix of the form

$$T = \begin{pmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{pmatrix},$$

where $T_{ij} \in \mathcal{L}(\mathcal{B}_j, \mathcal{B}'_i)$, $i, j = 1, 2$. (By $\mathcal{L}(\mathcal{B}_j, \mathcal{B}'_i)$ we denote the space of bounded linear operators acting from $\mathcal{B}_j$ to $\mathcal{B}'_i$ $(i, j = 1, 2)$.) The map G_T from $\mathcal{L}(\mathcal{B}_1, \mathcal{B}_2)$ into the set of closed affine subspaces of $\mathcal{L}(\mathcal{B}'_1, \mathcal{B}'_2)$, defined by

$$G_T(X) = \{Y \in \mathcal{L}(\mathcal{B}'_1, \mathcal{B}'_2) : \ T_{12} + T_{22}X = Y(T_{11} + T_{12}X)\}$$

is called a *linear fractional relation* (LFR) (associated with T).

Such relations can be considered as a generalization of linear fractional transformations which were studied by many authors and found many applications. In this talk we continue the study of LFR, the talk is devoted mostly to analogous of the Liouville theorem "a bounded entire function is constant" for LFR.