Spectral Estimates for Schrödinger Operator with Periodic Matrix Potentials on the Real Line

E. Korotyaev

We consider the Schrödinger operator on the real line with a $N \times N$ matrix valued periodic potential, $N > 1$. The spectrum of this operator is absolutely continuous and consists of intervals separated by gaps. We define the Lyapunov function, which is analytic on an associated N-sheeted Riemann surface. On each sheet the Lyapunov function has the standard properties of the Lyapunov function for the scalar case. The Lyapunov function has (real or complex) branch points, which we call resonances. We determine the asymptotics of the periodic, anti-periodic spectrum and of the resonances at high energy (in terms of the Fourier coefficients of the potential). We show that there exist two types of gaps:

i) stable gaps, i.e., the endpoints are periodic and anti-periodic eigenvalues,

ii) unstable (resonance) gaps, i.e., the endpoints are resonances (real branch points).

Moreover, the following results are obtained:

1) we define the quasimomentum as an analytic function on the Riemann surface of the Lyapunov function; various properties and estimates of the quasimomentum are obtained,

2) we construct the conformal mapping with real part given by the integrated density of states and imaginary part given by the Lyapunov exponent. We obtain various properties of this conformal mapping, which are similar to the case N=1,

3) we determine various new trace formulae for potentials and the Lyapunov exponent,

4) a priori estimates of gap lengths in terms of potentials are obtained.