Similarity of some Indefinite Sturm-Liouville Operators to Self-Adjoint Operators

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Let $\omega(\cdot) : \mathbb{R} \to \mathbb{R}_+$ be a positive measurable function on $\mathbb{R}$ and let $L^2(\mathbb{R},\ \omega)$ stand for the weighted Hilbert space consisting of measurable functions f on $\mathbb{R}$ that satisfy $\int_{\mathbb{R}} |f(t)|^2 \omega(t)dt < \infty$.

(i) Consider an operator

$$L_c := \frac{\operatorname{sgn} x}{|x|^\alpha} \left(-\frac{d^2}{dx^2} + c\delta \right), \quad c \in \mathbb{C}, \qquad \alpha > -1.$$

acting in $L^2(\mathbb{R},\ |x|^\alpha)$. Here δ is the Dirac delta. It is called an operator with a point interaction at zero.

We find a criterion for the operator L_c to be similar to a normal (self-adjoint) operator in terms of the interaction c.

(ii) Let $\omega(x) = p(x)|x|^\alpha, (\alpha > -1)$, and $0 < d \le p(x) \le D < \infty$. We find some sufficient conditions for an J–selfadjoint operator

$$L_\omega := -\frac{\operatorname{sgn} x}{p(x)|x|^\alpha} \frac{d^2}{dx},$$

in $L^2(\mathbb{R},\ \omega)$ to be similar to a selfadjoint operator. These conditions are expressed in terms of a function $p(\cdot)$.

The method of investigation of critical points of such type a definitizable operator is based on the Naboko–Malamud resolvent similarity criterion. Moreover, we exploit the theory of boundary triplets of symmetric operators and the corresponding Weyl functions.