On the Eigenvalues of Self-Adjoint Extensions with Exit

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Let A be a symmetric operator in a Krein space $\mathcal{K}$ and suppose that it has a self-adjoint extension A_0 in $\mathcal{K}$ that locally in some domain Ω it behaves like a self-adjoint relation in a Pontryagin space (that is, A is of type π_+ over Ω). Then we are interested in the eigenvalues of self-adjoint extensions $\widetilde{A}$ in a larger space $\widetilde{K} \supset \mathcal{K}$, under the assumption that $\widetilde{A}$ also is of type π_+ over Ω.

We are giving an analytic characterization in terms of the functions $m(\lambda)$, which is the Q- (or Weyl) function of the pair (A, A_0), and the parameter $\tau(\lambda)$ in

$$P_{\mathcal{K}}\big(\widetilde{A} - \lambda\big)^{-1}|_{\mathcal{K}} = (A_0 - \lambda)^{-1} - \gamma(\lambda)\big(m(\lambda) + \tau(\lambda)\big)^{-1}\gamma(\overline{\lambda})^{+},$$

where $\gamma(\lambda)$ denotes the corresponding γ-field.

In particular, it follows that $\widetilde{A}$ in general is not a minimal representing relation for the function $-\frac{1}{m(\lambda)+\tau(\lambda)}$ as it is in the case of canonical extensions, where τ is a constant. Furthermore, we give a necessary condition on the spectral measure of A_0 for the existence of embedded eigenvalues for some extension $\widetilde{A}$.