Around Models of $\mathcal{N}_\kappa^\infty$–Functions

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Each generalized Nevanlinna functions $N(z) \in \mathcal{N}_\kappa$ for which $z = \infty$ is the only pole (or generalized pole) of non-positive type is characterized by the *irreducible* representation

$$N(z) = (z - z_0)^m (z - z_0^*)^m (N_0(z) + q(z)) + p(z), \tag{1}$$

where z_0 belongs to the domain of holomorphy of N, $m \in \mathbb{N}_0$, $N_0(z) \in \mathcal{N}_0$, $q(z)$ and $p(z)$ are real polynomials, $\deg p \leq 2m - 1$. It can be shown that $N(z)$ in (1) admits also another representation

$$N(z) = (z - z_0)^\kappa (z - z_0^*)^\kappa N_0^r(z) + r(z), \tag{2}$$

with some $N_0^r(z) \in \mathcal{N}_0$ and a real polynomial $r(z)$ of degree $\leq 2\kappa - 1$. In general the representation (2) is *reducible*. Earlier we have described minimal representations of $N(z)$ and $-N(z)^{-1}$ in the reproducing kernel space $\mathcal{L}(N_0) \oplus \mathcal{L}(q) \oplus \mathcal{L}(M_p)$ associated with the factors N_0, q and $((z - z_0)^m, p)$ of the irreducible representation (1). Here we consider models of $N(z)$ and $-N(z)^{-1}$ in the reproducing kernel space $\mathcal{L}(N_0^r) \oplus \mathcal{L}(M_r)$ associated with the factors N_0^r and $((z - z_0)^\kappa, r)$ of the representation (2) and also a correspondence between the models. The meaning of representation (2) and associated models for an approximation problem will be discussed.