Extension of Symmetric Operators with Finitely Many Negative Squares in Krein Spaces

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Let S be a simple symmetric operator of defect one in a Krein space $\mathcal{K}$ and assume that S has finitely many negative squares and a canonical self-adjoint extension with a nonempty resolvent set. We investigate the number of negative squares of self-adjoint extensions $\widetilde{A}$ of S which act in a larger space $\mathcal{K} \times \mathcal{H}$.

In particular we also obtain a characterization of the number of negative squares of the canonical self-adjoint extensions of a simple symmetric operator S of defect one with finitely many negative squares.

The main tool are functions belonging to a so-called class D_κ, which is a subclass of the definitizable functions. We say that a function τ, meromorphic in $\mathbb{C}\backslash\mathbb{R}$, symmetric with respect to $\mathbb{R}$ and holomorphic at λ_0 belongs to the class D_κ, $\kappa \in \mathbb{N}_0$, if there exists a generalized Nevanlinna function $G \in N_\kappa$ holomorphic at λ_0 and a rational function g holomorphic in $\overline{\mathbb{C}}\backslash\{\lambda_0, \overline{\lambda}_0\}$ such that

$$\frac{\lambda}{(\lambda - \lambda_0)(\lambda - \overline{\lambda}_0)}\tau(\lambda) = G(\lambda) + g(\lambda)$$

holds for all points λ where τ, G and g are holomorphic.