Indefinite Sturm-Liouville Operators with Singular Self-Similar Weights

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We study the asymptotics of the spectrum for the boundary eigenvalue problem

\[-y'' - \lambda \rho y = 0,\]
\[y(0) = y(1) = 0,\]

where \(\rho \in W^{-1}_2[0, 1]\) is the generalized derivative of fractal (self-similar) function \(P \in L^2[0, 1]\).

We prove that the eigenvalues of positive and negative type accumulate to ±∞ and their counting functions have representation

\[N_\pm(\lambda) = |\lambda|^{D/2} \cdot (s_\pm (\ln |\lambda|) + o(1)), \quad |\lambda| \to \infty \quad (**),\]

where \(D \in (0, 2)\) is the fractal dimension of the function \(P\) and \(s_\pm\) are some periodic functions. We study three cases of self-similarity of \(P\) which induce different behaviour of functions \(s_\pm\).

The particular case of definite \(\rho\), when \(\rho\) is a self-similar measure, was studied by M. Solomyak and E. Verbitsky. (A paper of M. Levitin and D. Vassiliev is also related to the topic). They obtained the asymptotic formula (** with \(s_- \equiv 0\) and \(D \in (0, 1]\).

We pay attention that fractal dimension \(D\) can be greater than 1 only in the case when \(\rho\) is indefinite.

The talk is based on the joint works with A.A. Vladimirov: