The Similarity Problem for $J$-Nonnegative Sturm-Liouville Operators

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We present new sufficient conditions for the similarity of $J$-self-adjoint Sturm-Liouville operators to self-adjoint ones. These conditions are formulated in terms of Weyl-Titchmarsh $m$-coefficients. This result is exploit to prove the regularity of the critical point zero for various classes of $J$-nonnegative Sturm-Liouville operators. In particular, we prove that 0 is a regular critical point of

$$A = (\text{sgn } x)(-d^2/dx^2 + q(x))$$

if \( q \in L^1(\mathbb{R}, (1 + |x|)dx) \). Moreover, in this case $A$ is similar to a self-adjoint operator if and only if it is $J$-nonnegative. We also show that the latter condition on $q$ is sharp, that is we construct a potential \( q_0 \in \cap_{\gamma < 1} L^1(\mathbb{R}, (1 + |x|^\gamma)dx) \) such that the operator $A$ is $J$-nonnegative with the singular critical point zero and hence is not similar to a self-adjoint one.