Some inverse problems for operator-differential equations of mixed type

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We consider here the following inverse problem. Find a function $u(t)$ and an element φ satisfying the equation

$$Bu_t(t) + Lu(t) = \gamma(t)\varphi + f(t),$$

$$u(0) = u_0, \qquad u(T) = u_T.$$

Operators B and L are selfadjoint in a Hilbert space E; the spectrum of the operator L is semibounded; $\gamma(t)$ is a scalar function. It is proved that if a finite number of orthogonality conditions holds then the inverse problem is uniquely solvable. The method uses the representation as a series in eigenelements and associated elements of the pencil $L - \lambda B$.

Moreover we consider the integro-differential equation

$$Bu_t(t) + Lu(t) = \int_0^t k(t-s)Lu(s)ds + Bg(t), \qquad t \in (0,T),$$

where the scalar kernel $k : [0,T] \to \mathbb{C}$ is unknown. We identify a convolution kernel k — i.e. we prove existence and uniqueness of k — in a first-order singular integro-differential operator equation of Volterra type in two overdetermined problems in the framework of Hilbert spaces. We stress that in the first problem, by virtue of specific nonlocal time conditions, the kernel can be recovered globally in time, while in the latter only locally in time because of conditions that are of time-periodic type.