Conservative discrete time-invariant systems and block operator CMV matrices

Yu. Arlinskiĭ

It is well known that an operator-valued function Θ from the Schur class $\mathbf{S}(\mathfrak{M}, \mathfrak{N})$, where $\mathfrak{M}$ and $\mathfrak{N}$ are separable Hilbert spaces, can be realized as the transfer function of a simple conservative discrete time-invariant linear system. The known realizations involve the function Θ itself, the Hardy spaces or the reproducing kernel Hilbert spaces. On the other hand, as in the classical scalar case, the Schur class operator-valued function is uniquely determined by its so called "Schur parameters". In this talk we present simple conservative realizations of an operator-valued Schur class function using its Schur parameters only. It turns out that the unitary operators corresponding to the systems take the form of five-diagonal block operator matrices, which are the analogs of Cantero–Moral–Velázquez (CMV) matrices appeared recently in the theory of scalar orthogonal polynomials on the unit circle. For an arbitrary completely non-unitary contraction we obtain new models given by truncated block operator CMV matrices. We show that the minimal unitary dilations of a contraction in a Hilbert space and the minimal Naimark dilations of a semi-spectral operator measure on the unit circle can also be expressed by means of block operator CMV matrices.