Quadratic (weakly) hyperbolic matrix polynomials: Inverse spectral problems

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joint work with A. Dijksma, K.-H. Förster, and P. Jonas

The main result of the talk is the following theorem: *Let* n *be an integer* ≥ 2 *and assume that the ordered set* $\{\beta_{\pm j}\}_{j=1}^{n-1} \in T_{2n-2}$ *block–interlaces the ordered set* $\{\alpha_{\pm j}\}_{j=1}^{n} \in T_{2n}$*. Then there exist* $n \times n$ *Jacobi matrices* B *and* C *such that*

(i) the matrix polynomial $L(\lambda) = \lambda^2 + \lambda B + C$ *is weakly hyperbolic,*

(ii) the ordered eigenvalues of L *coincide with* $\{\alpha_{\pm j}\}_{j=1}^{n}$*, and*

(iii) the ordered eigenvalues of the compression $L_{\infty;e_n}$ *of* L *to* $\{e_n\}^{\perp}$ *with* $e_n = \begin{pmatrix} 0 & \cdots & 0 & 1 \end{pmatrix}^{\top} \in \mathbb{C}^n$ *coincide with* $\{\beta_{\pm j}\}_{j=1}^{n-1}$*.*

If, in addition,

$$\alpha_1 - \alpha_{-1} > 0, \tag{1}$$

then L *is hyperbolic.*

The lecture is based on joint work with Aad Dijksma, Karl-Heinz Foerster, and Peter Jonas started in 2001, but just recently finished. In another lecture Aad Dijksma will discuss a direct spectral problem.

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