Inverse spectral and scattering theory for the half-line left-definite Sturm-Liouville problem

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The problem of integrating the Camassa-Holm equation leads to the scattering and inverse scattering problem for the Sturm-Liouville equation $-u'' + \frac{1}{4}u = \lambda w u$ where w is a weight function which may change sign but where the left hand side gives rise to a positive quadratic form so that one is led to a left-definite spectral problem.

In this talk the spectral theory and a generalized Fourier transform associated with the equation $-u'' + \frac{1}{4}u = \lambda w u$ posed on a half-line are investigated. An inverse spectral theorem and an inverse scattering theorem are established. A crucial ingredient of the proofs of these results is a theorem of Paley-Wiener type.