On computing the instability index of a non-selfadjoint differential operator associated with coating and rimming flows

A. Burchard joint work with M. Chugunova

We study the problem of finding the instability index of certain non-selfadjoint fourth order differential operators that appear as linearizations of coating and rimming flows, where a thin layer of fluid coats a horizontal rotating cylinder. The main result reduces the computation of the instability index to a finite-dimensional space of trigonometric polynomials. The proof uses Lyapunov's method to associate the differential operator with a quadratic form, whose maximal positive subspace has dimension equal to the instability index. The quadratic form is given by a solution of Lyapunov's equation, which here takes the form of a fourth order linear PDE in two variables. Elliptic estimates for the solution of this PDE play a key role. We include some numerical examples.