On the uniform convergence of diagonal Padé approximants

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Let $d\sigma$ be a finite nonnegative measure on $E = [-1, \alpha] \cup [\beta, 1]$ and let

$$\mathfrak{F}(\lambda) = \int_E \frac{t d\sigma(t)}{t - \lambda}.$$

As was shown by H. Stahl (1983) there exists a function $\mathfrak{F}_0$ of the above described type with $\alpha = \beta$ such that the diagonal Pade approximants for $\mathfrak{F}_0$ do not converge on $\mathbb{R}$. In our work, it is shown that there is a subsequence of the diagonal Padé approximants for $\mathfrak{F}$, which converges locally uniformly to $\mathfrak{F}$ in the gap (α, β). Moreover, we present the necessary and sufficient condition of the existence of a subsequence of the diagonal Padé approximants for $\mathfrak{F}$, which converges locally uniformly to $\mathfrak{F}$ in $\mathbb{C} \setminus ([-1-\varepsilon, \alpha] \cup [\beta, 1+\varepsilon])$ for some $\varepsilon > 0$. Convergence results for some larger classes of meromorphic functions are also considered.

This talk is a continuation of the talk given by Vladimir A. Derkach at the last workshop.