On linear fractional transformations associated with generalized J-inner matrix functions

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joint work with H. Dym

We study generalized J-inner matrix valued functions $W(\lambda)$ decomposed in the block form $W(\lambda) = \begin{pmatrix} w_{11} & w_{12} \\ w_{21} & w_{22} \end{pmatrix}$ conformally with $J = \begin{pmatrix} I_p & 0 \\ 0 & -I_q \end{pmatrix}$, which appear as resolvent matrices in various indefinite interpolation problems. Reproducing kernel indefinite inner product spaces associated with a generalized J-inner matrix valued function $W(\lambda)$ are studied and intensively used in the description of the range of the linear fractional transformation $T_W[\varepsilon] = (w_{11}\varepsilon + w_{12})(w_{21}\varepsilon + w_{22})^{-1}$ applied to the Schur class $S^{p\times q}$. For a subclass $\mathcal{U}_\kappa^\circ(J)$ of generalized J-inner matrix valued function W the notion of associated pair is introduced and factorization formulas for W are found. These results are used in order to describe the set of generalized Schur functions from $T_W[S^{p\times q}]$ with maximal negative signature.