Completely bounded kernels

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Given a set X and two C^*-algebras $\mathcal{A}$ and $\mathcal{B}$, a kernel k is defined as a function from $X \times X$ to $L(\mathcal{A}, \mathcal{B})$, the bounded linear maps from $\mathcal{A}$ to $\mathcal{B}$. The kernel k is *positive* if for all finite sets $F = \{(x_j, a_j)\} \subset X \times \mathcal{A}$, the matrix

$$\big(k(x_i, x_j)[a_i a_j^*]\big)_{F \times F} \qquad (*)$$

is nonnegative. If the same is true whenever we replace $X \times \mathcal{A}$ by $X \times M_n(\mathcal{A})$ and k by $k \otimes 1_n$ for any $n \in \mathbb{N}$, then k is said to be *completely positive* (the two concepts coincide when $\mathcal{A} = \mathcal{B} = \mathbb{C}$). Completely positive kernels have several equivalent characterisations, including the existence of a so-called Kolmogorov decomposition. Constantinescu and Gheondea, generalising results of Laurent Schwarz, considered kernels k where the matrix in $(*)$ is merely selfadjoint with $L(\mathcal{A}, \mathcal{B}) = B(\mathcal{H})$, $\mathcal{H}$ a Hilbert space, and found necessary and sufficient conditions for the decomposability of such kernels as the difference of (completely) positive kernels. A result of Haagerup implies that when $\mathcal{A}$ and $\mathcal{B}$ are von Neumann algebras such decompositions in terms of completely positive kernels will fail if $\mathcal{B}$ is not injective.

In this talk we discuss decomposability of self adjoint kernels as differences of completely positive kernels when $\mathcal{A}$ and $\mathcal{B}$ are C^*-algebras, characterising decomposable kernels. We also discuss the case when the matrix in $(*)$ is a only a completely bounded map, giving an analogue of the Wittstock decomposition for such kernels.