Two models of Krein-space related physics: the MHD α^2-dynamo and the $\mathcal{PT}-$symmetric Bose-Hubbard model

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Two simple physical models are discussed whose operators are selfadjoint in Krein-spaces.

In the first part of the talk, the eigenvalue behavior $\lambda(\alpha, \beta)$ of the 2×2 matrix differential operator of the spherically symmetric α^2-dynamo of magnetohydrodynamics is considered for constant $\alpha-$profiles and boundary conditions which depend on a parameter β. Specifically, $\beta \in [0, 1]$ acts as parameter in the homotopic interpolation between idealized (Dirichlet) and physically realistic (Robin) boundary conditions (BCs). For the quasi-exactly solvable monopole setup (with spherical mode number $l = 0$) the characteristic equation is derived explicitly. It is shown that the $\beta-$homotopy describes an interpolation between spectra of mesh type (idealized BCs) and a countably infinite set of parabolas (physically realistic Robin Bcs). Interestingly, the mesh nodes (semisimple twofold degenerate eigenvalues) are fixed points of the $\beta-$homotopy. An underlying ruled-surface structure of the spectrum is uncovered.

In the second part of the talk, we provide a brief summary of recent results on the spectral behavior of the $\mathcal{PT}-$symmetric Bose-Hubbard system as it is used for the description of quantum Bose-Einstein condensates with balanced gain-loss interactions. For an $N-$particle system the corresponding Fock-space Hamiltonian reduces to an $N \times N-$matrix which is selfadjoint in an $N-$dimensional Pontryagin space. The unfolding of higher-order branch-points of the spectrum is considered under parameter perturbations. Numerical as well as analytical results are presented which demonstrate the relevance of the Hessenberg type of the Hamiltonian as defining matrix structure for the occurrence of specific Galois cycles in the eigenvalue rings of the unfolding branch points.

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