On interconnection of conservative systems

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We show that a large class of conservative systems can be characterised by the fact that their system variables live on a Lagrangian subspace of a certain Kreĭn space $\mathfrak{K}$. Most relevant Lagrangian subspaces of $\mathfrak{K}$ also arise from conservative linear systems.

Energy-preserving interconnection of two finite-dimensional conservative systems is a conservative system. However, in infinite dimensions conservativity is not always preserved under energy-preserving interconnection. We show that this interconnection problem for conservative systems leads to the following *abstract compression problem*:

Let $\mathfrak{K}_r$ and $\mathfrak{K}_d$ be Kreĭn spaces with indefinite inner products $[\cdot,\cdot]_{\mathfrak{K}_r}$ and $[\cdot,\cdot]_{\mathfrak{K}_d}$, respectively. Let $\mathfrak{K}$ be the Kreĭn space $\left[\begin{smallmatrix}\mathfrak{K}_r\\ \mathfrak{K}_d\end{smallmatrix}\right]$ with inner product

$$\left[\begin{bmatrix} k_r \\ k_d \end{bmatrix}, \begin{bmatrix} k_r' \\ k_d' \end{bmatrix}\right]_{\mathfrak{K}} = [k_r, k_r']_{\mathfrak{K}_r} + [k_d, k_d']_{\mathfrak{K}_d}.$$

Let V be a Lagrangian subspace of $\mathfrak{K}$ and let $G \subset \mathfrak{K}_d$. The problem is to find necessary and sufficient conditions on V and G for the compression

$$V_r := \left\{ k_r \in \mathfrak{K}_r \mid \exists k_d \in G : \begin{bmatrix} k_r \\ k_d \end{bmatrix} \in V \right\}$$

to be a Lagrangian subspace of $\mathfrak{K}_r$.

The approach might depend on the particular properties of $\mathfrak{K}$ and V. We provide a full abstract solution and some more practical partial solutions.