Schur algorithm for generalized Caratheodory functions

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We define the Schur algorithm for generalized Caratheodory functions and study its properties.

The function $f(z)$ is called a *generalized Caratheodory function with $\varkappa$ negative squares* if it is meromorphic in the open unit disc $\mathbb{D}$ and the kernel

$$K_f(z,w) = \frac{f(z) + f(w)^*}{1 - zw^*}$$

has $\varkappa$ negative squares in the domain of holomorphy of $f(z)$ in $\mathbb{D}$. We denote this class of functions which are holomorphic at $z_1 \in \mathbb{D}$ by $\mathbf{C}_{\varkappa}^{z_1}$.

Theorem. *Let $f \in \mathbf{C}_{\varkappa}^{z_1}$ has the Taylor expansion*

$$f(z) = \sum_{i=0}^{\infty} c_i (z - z_1)^i$$

and let $f_1(z)$ be the Schur transformation of $f(z)$. Then $f_1 \in \mathbf{C}_{\varkappa_1}^{z_1}$, where

if $Re\, c_0 \neq 0$ and $c_0 + c_0^* > 0$, then $\varkappa_1 = \varkappa$

if $Re\, c_0 \neq 0$ and $c_0 + c_0^* < 0$, then $\varkappa_1 = \varkappa - 1$

if $Re\, c_0 = 0$, then $\varkappa_1 = \varkappa - k$, where $k \geqslant 1$ is the smallest integer such that $c_k \neq 0$.

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