Estimate of essential spectrum of Schrödinger operator with δ' perturbation supported by an asymptotically straight curve

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Perturbations of the Laplace operator by δ-potentials supported by a curve (leaky wires Hamiltonians) are studied in last decade. In particular, for an asymptotically straight curve Γ on $\mathbb{R}^2$ an estimate on the spectrum of the perturbation was obtained in the article "Conditions for the spectrum associated with a leaky wire to contain the interval $[-\frac{\alpha_0^2}{4}, +\infty)$" by Brown, B. Malcolm; Eastham, M.S.P.; Wood, Ian. We generalized the technique used in the work to obtain similar estimation for δ'-perturbation.

Consider the operator

$$H := -\Delta - \alpha(\mathbf{x})\delta'(\mathbf{x} - \Gamma).$$

Such an operator can be defined as a closure of the e.s.a. operator

$$\hat{H} = -\Delta\psi(\mathbf{x}).$$

with the domain consisting of functions $\psi \in H^2(\mathbb{R}^2)$ which satisfy δ' boundary conditions:

$$\left\{ \begin{array}{rcl} \frac{\partial\psi}{\partial n_+}(\mathbf{x}) + \frac{\partial\psi}{\partial n_-}(\mathbf{x}) & = & 0, \\ \psi_+(\mathbf{x}) - \psi_-(\mathbf{x}) & = & \alpha(\mathbf{x})\frac{\partial\psi}{\partial n_+}, \end{array} \right. \quad \mathbf{x} \in \Gamma,$$

where $\psi_\pm$ and $\frac{\partial\psi}{\partial n_\pm}$ denote one-side limits and normal derivatives of ψ.

Suppose $\alpha(\mathbf{x})$ tends sufficiently fast to a constant α_0 as $\mathbf{x} \to \infty$. Then we prove that $[-\frac{4}{\alpha_0^2}, +\infty) \subset \sigma_{\text{ess}}$ under certain conditions on the curve Γ.