Eigenvalues in spectral gaps of J-selfadjoint operators and indefinite Sturm-Liouville operators

R. Möws
joint work with J. Behrndt and C. Trunk

Consider two J-selfadjoint operators A and B with $\rho(A) \cap \rho(B) \neq \emptyset$, which are a one-dimensional perturbation in the resolvent sense of each other, i.e.

$$\dim \operatorname{ran}((A-\lambda)^{-1} - (B-\lambda)^{-1}) = 1$$

for $\lambda \in \rho(A) \cap \rho(B)$.

Assume that B has κ negative squares and there exists some inverval $I \subset \rho(B) \cap \mathbb{R}$. We show that $\sigma(A) \cap I$ consists only of at most finitely many eigenvalues. Furthermore, we give an upper bound on the number of eigenvalues of A in I depending only on κ.

This result can be applied to a $J-$selfadjoint operator A associated to the singular indefinite Sturm-Liouville expression

$$\operatorname{sgn}(-f'' + qf),$$

defined on $\mathbb{R}$, where $q \in L^1_{\text{loc}}(\mathbb{R})$. Assume that the limits $q_\infty = \lim_{x\to\infty} q(x)$ and $q_{-\infty} = \lim_{x\to-\infty} q(x)$ exist and fulfill $-q_{-\infty} < q_\infty$. Then $(-q_{-\infty}, q_\infty)$ is a gap in the essential spectrum of A. We will give an estimate for the number of eigenvalues of A in $(-q_{-\infty}, q_\infty)$.