On the unitary equivalence of absolutely continuous parts of self-adjoint extensions

H. Neidhardt

The classical Weyl-Neumann theorem states that for any selfadjoint operator A on a Hilbert space there exists a (non-unique) Hilbert-Schmidt operator $B = B^*(\in \mathfrak{S}_2)$ such that the perturbed operator $A+B$ has purely point spectrum. We are interesting whether this result remains valid for non-additive perturbations by considering self-adjoint extensions of a given densely defined symmetric operator A in $\mathfrak{H}$ and fixing an extension $A_0 = A_0^*$. We show that for a wide class of symmetric operators the absolutely continuous parts of extensions $\widetilde{A} = \widetilde{A}^*$ and A_0 are unitarily equivalent provided that their resolvent difference is a compact operator. Namely, we show that this property holds true whenever a Weyl function $M(\cdot)$ of a pair $\{A, A_0\}$ satisfies the following property: the limit $M(x) := s - \lim_{y\to+0} M(x+iy)$ exists and is bounded for a. e. $x \in \mathbb{R}$. This result is applied to some direct sums of symmetric operators.