Dilations and passive impedance optimal realizations of Caratheodory class operator-valued functions

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Let $\ell(U)$ be the class of all Caratheodory functions (analytic inside open unit disc with nonnegative real part) whose values are bounded linear operators mapping separabel Hilbert space U into U. In the development of the Darlington method for passive linear time-invariant input/state/output systems (by Arov, Dewilde, Douglas and Helton) the following question arose: do there exist simple necessary and sufficient conditions under wich a function $c \in \ell(U)$ has a (J_1, J_2)-bi-inner dilation θ mapping Y_1 into Y_2; here Y_1 and Y_2 are two separabel Hilbert spaces such that $U \subset Y_1$, $U \subset Y_2$, and the requirement that θ is (J_1, J_2)-bi-inner means that θ is analytic and (J_1, J_2)-bi-contractive in open unit disc and has (J_1, J_2)-unitary nontangential limits a.e. on unit circle. We prove that there are two necessary and sufficient conditions of existing of such a dilation: 1) factorization equations

$$\varphi(\zeta)^*\varphi(\zeta) = 2\Re c(\zeta), \qquad \psi(\zeta)\psi(\zeta)^* = 2\Re c(\zeta), \quad \text{a.e. } |\zeta| = 1,$$

have nonzero solutions φ and ψ in classes of analytic inside open unit disc operator-valued fuctions; 2) scattering suboperator $s_c(\zeta)$ of function c has a denominator $\{b_1, b_2\}$. We discribe the set of all dilations of function $c \in \ell(U)$. Also we prove that $c \in \ell(U)$ has a (J_1, J_2)-bi-inner minimal and optimal (minimal and *-optimal) dilation θ if and only if the minimal and optimal (minimal and *-optimal) passive impedance realization of c is strongly bi-stable.

Mentioned above results can be found in [1], [2].

References

[1] Arov D.Z., Rozhenko N.A. To the theory of passive impedance systems with lossess of scattering channels // *Zapiski Nauchnykh Seminarov POMI, Saint-Peterburg. – 2008. – Vol. 355. – P. 37-71.*

[2] Arov D., Dym H. J-contractive matrix-valued functions and related topics. – *Cambrige University Press, 2008. – 575 pp.*