Properties of nonlinear maps associated with inverse Sturm-Liouville problems

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Denote by L_D the operator generated by the Sturm-Liouville differential expression $Ly = -y'' + q(x)y$ and the Dirichlet boundary conditions at the finite interval $[0, 1]$. We assume that $q(x)$ belongs to the Sobolev space $W_2^\theta[0, 1]$ with some $\theta \geqslant -1$. The classical inverse problem for this operator is formulated as follows: to recover the potential $q(x)$ by the given spectral function of L_D which is defined by the spectrum $\{\lambda_k\}_1^\infty$ and the so-called norming constants $\{\alpha_k\}_1^\infty$. These two sequences are called the spectral data of L_D.

For given $\theta \geqslant -1$ we construct special Hilbert space (denoted by $\hat{l}_2^\theta$) where the spectral data are placed in when the potential q runs through the Sobolev space $W_2^\theta[0, 1]$. Then we study the maps $F : q \to \eta = \{\lambda_k, \alpha_k\}_1^\infty$ acting from W_2^θ to $\hat{l}_2^\theta$ and show that for any $\theta > -1$ the map F is is weakly nonlinear, i.e. a compact perturbation of a linear map.

The main result (which is new in classical case, too) roughly can be formulated as follows: if η and $\tilde{\eta}$ are the vectors characterizing the spectral data of the potentials q and $\tilde{q}$, respectively, then the difference $q - \tilde{q}$ in the norm of W_2^θ can be uniformly estimated through the difference $\eta - \tilde{\eta}$ in the norm $\hat{l}_2^\theta$.