

How to complete a maximal nonnegative subspace of a Krein space?

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Let Z be a maximal nonnegative subspace of a Krein space K , let Z^\perp be the orthogonal companion to Z in K , and let $Z_0 = Z \cap Z^\perp$ be the maximal neutral subspace of Z . Then the quotient spaces Z/Z_0 and Z^\perp/Z_0 inherit positive inner products from K and $-K$, respectively. The topologies induced by these two inner product spaces are not, in general, complete. We show that the completions of the spaces Z/Z_0 and Z^\perp/Z_0 with these inner products can be identified in a natural way with certain subspaces of the quotient spaces K/Z^\perp and K/Z , respectively. The construction of these subspaces is similar to the de Brange-Rovnyak construction used to realize an operator-valued Schur function in the unit disk D as the characteristic function of a discrete time input/state/output system.