p-Adic Schrödinger-type operator with point interactions

S. Torba
joint work with S. Albeverio and S. Kuzhel

The function calculus of functions acting on the field $\mathbb{Q}_p$ of p-adic numbers with values in $\mathbb{C}$ is considered. There are analogues of integral, scalar product, L_2-space and Fourier transform in this calculus, but no derivative. So an operator of fractional differentiation D^α of order $\alpha > 0$ plays a corresponding role. p-adic Schrödinger-type operators with potentials $V(x) : \mathbb{Q}_p \to \mathbb{C}$ are defined as $D^\alpha + V(x)$. In this talk, finite rank point perturbations are considered. General expression of such perturbation is $V_Y = \sum_{i,j=1}^n b_{ij}\langle\delta_{x_j}, \cdot\rangle\delta_{x_i}$, where δ_x is the Dirac delta function and $\{x_i\}_{i=1}^n$ are some p-adic points. Operator realizations of $D^\alpha + V_Y$ in $L_2(\mathbb{Q}_p)$ are described. Such problem is well-posed for $\alpha > 1/2$ and the singular perturbation V_Y is form-bounded for $\alpha > 1$. Spectral properties of operator realizations are studied, and the corresponding Krein's resolvent formula is given.

Let η be an invertible bounded self-adjoint operator in $L_2(\mathbb{Q}_p)$. An operator A is called *η-self-adjoint* in $L_2(\mathbb{Q}_p)$ if $A^* = \eta N \eta^{-1}$. η-self-adjoint operator realizations of $D^\alpha + V_Y$ in $L_2(\mathbb{Q}_p)$ for $\alpha > 1$ are described, and each realization is given in the form of some boundary valued space.

Each η-self-adjoint operator A is self-adjoint in a Krein space $(L_2(\mathbb{Q}_p), [\cdot,\cdot])$ with indefinite metric $[f, g] = (\eta f, g)$. To overcome difficulties of dealing with the indefinite metric, the hidden symmetry of operator A that is represented by the linear operator $\mathcal{C}$ is considered. Remind that an η-self-adjoint operator A has the property of *$\mathcal{C}$-symmetry* if there exists a bounded linear operator $\mathcal{C}$ in $L_2(\mathbb{Q}_p)$, such that (i) $\mathcal{C}^2 = I$; (ii) $\eta\mathcal{C} > 0$; (iii) $A\mathcal{C} = \mathcal{C}A$. It is proven that if $A_\mathcal{B}$ is the η-self-adjoint operator realization of $D^\alpha + V_Y$, then the following statements are equivalent: (i) $A_\mathcal{B}$ possesses the property of $\mathcal{C}$-summetry; (ii) the spectrum $\sigma(A_\mathcal{B})$ is real and there exists a Rietz basis of $L_2(\mathbb{Q}_p)$ composed of eigenfunctions of $A_\mathcal{B}$.