On $\mathcal{PT}$ symmetric operators

C. Trunk
joint work with T. Azizov

We consider so-called $\mathcal{PT}$ symmetric operators in the Krein space $(L_2(\mathbb{R}), [\cdot,\cdot])$, where $[\cdot,\cdot]$ is given via the fundamental symmetry $\mathcal{P}f(x) = f(-x)$. The action of the anti-linear operator $\mathcal{T}$ on a function of a real spatial variable x is defined by $\mathcal{T}f(x) = \overline{f(x)}$, and thus $\mathcal{T}^2 = I$ and $\mathcal{PT} = \mathcal{TP}$ follow. An operator A is said to be $\mathcal{PT}$-symmetric if it commutes with $\mathcal{PT}$.

In the last decade the following operator defined via the differential expression

$$(\tau y)(x) := -y''(x) + x^2(ix)^\epsilon y(x), \quad \epsilon > 0$$

was studied intensively.

We will start our investigations with the discussion of the case ϵ is even. In this case we give a full description of the spectral properties and of all boundary conditions which lead to $\mathcal{PT}$ symmetric operators. Further results are obtained via the perturbation theory for self-adjoint operators in Krein spaces.