Friday, December 16th

9:00 – 9:10 *Opening*

Chair: Henk de Snoo

9:10 – 9:35 **Heinz Langer**
Sturm-Liouville operators with singularities and generalized Nevanlinna functions

9:35 – 10:00 **Leiba Rodman**
Canonical forms for pairs of symmetric, skew symmetric, or hermitian matrices: Some applications and open problems

10:00 – 10:25 **Manfred Möller**
A completeness theorem for a non-standard two-parameter eigenvalue problem

10:25 – 10:50 **Annemarie Luger**
On the eigenvalues of self-adjoint extensions with exit

10:50 – 11:45 *Refund of travel expenses (MA 674)*

& *Coffee break (DFG Lounge MA 315)*

Friday, December 16th

Chair: Manfred Möller

11:45 – 12:10 **Sergei G. Pyatkov**
Some properties of eigenfunctions and associated functions of indefinite Sturm-Liouville problems

12:10 – 12:35 **Mark Malamud**
Weyl functions and generalization of the Aronszajn-Donoghue theory

12:35 – 13:00 **Andrei Shkalikov**
PT- and CPT-symmetric operators. Critical parameter values. Spectral portraits.

13:00 – 13:25 **Matthias Langer**
Variational principles for eigenvalues of the Klein-Gordon equation

13:25 – 15:15 *Lunch break*

Friday, December 16th

Chair: Aad Dijksma

15:15 – 15:40 **Tomas Azizov**
On a generalization of a Pontryagin space property

15:40 – 16:05 **Yury M. Arlinskii**
Non-self-adjoint Jacobi matrices with rank one imaginary part

16:05 – 16:30 **Rostyslav Hryniv**
Inverse spectral problems for Bessel operators

16:30 – 16:55 **Evgeny Korotyaev**
Spectral estimates for Schrödinger operators with periodic matrix potentials on the real line

16:55 – 17:40 *Coffee break (DFG Lounge MA 315)*

Friday, December 16th

Chair: Andreas Lasarow

17:40 – 18:05 **Adrian Sandovici**
Extension theory of sectorial linear relations. A factorization approach

18:05 – 18:30 **Friedrich Philipp**
Spectral points of type π_+ of symmetric operators in indefinite inner product spaces

18:30 – 18:55 **Andreas Fleige**
A necessary aspect of the generalized Beals condition for the Riesz basis property of indefinite Sturm-Liouville problems

20:00 *Conference dinner*

SARAY, Uhlandstraße 142, 10719 Berlin

Saturday, December 17th

Chair: Henrik Winkler

9:00 – 9:25 **Henk de Snoo**
Limit-point/limit-circle classification for Sturm-Liouville problems whose coefficients depend rationally on the eigenvalue parameter

9:25 – 9:50 **Christiane Tretter**
The spectra of multiplication operators associated with families of operators

9:50 – 10:15 **Harald Woracek**
Shifted Hermite-Biehler functions

10:15 – 10:40 **Vyacheslav Pivovarchik**
Shifted Hermite-Biehler functions II. Applications

10:40 – 11:30 *Conference photo*

& *Coffee break (DFG Lounge MA 315)*

Saturday, December 17th

Chair: Annemarie Luger

11:30 – 11:55 **Andre C.M. Ran**
Semidefinite invariant subspaces for normal and hyponormal matrices in an indefinite inner product space

11:55 – 12:20 **Hagen Neidhardt**
Block matrices, boundary triplets and scattering

12:20 – 12:45 **Kresimir Veselic**
New perturbation estimates for eigenvalues and eigenvectors of selfadjoint operators

12:45 – 14:45 *Lunch break*

Saturday, December 17th

Chair: Sergei G. Pyatkov

14:45 – 15:10 **Aurelian Gheondea**
On Krein spaces induced by symmetric operators

15:10 – 15:35 **Victor Khatskevich**
An analogue of the Liouville theorem for linear relations in Banach spaces

15:35 – 16:00 **Ilia Karabash**
On eigenvalues of non-definitizable differential operators

16:00 – 16:25 **Daniel Alpay**
Discrete analogs of canonical systems with pseudo-exponential potential. Definitions and formulas for the spectral matrix functions

16:25 – 17:00 *Coffee Break (DFG Lounge MA 315)*

Saturday, December 17th

Chair: Vadim Adamyan

17:00 – 17:25 **Maxim Derevyagin**
On the uniform convergence of Pade approximants for generalized Nevanlinna functions

17:25 – 17:50 **Aleksey Kostenko**
Similarity of some indefinite Sturm-Liouville operators to self-adjoint operators

17:50 – 18:15 **Lyudmila I. Sukhocheva**
Quadratic operator pencils and selfadjoint operators in Krein space

Sunday, December 18th

Chair: Heinz Langer

9:00 – 9:25 **Vadim Adamyan**
Some class of solvable potentials for Dirac type systems

9:25 – 9:50 **Aad Dijksma**
On the Schur transformation of generalized Nevanlinna functions

9:50 – 10:15 **Birgit Jacob**
Minimum-phase infinite-dimensional second-order systems

10:15 – 10:40 **Carsten Trunk**
Extensions of symmetric operators with finitely many negative squares in Krein spaces

10:40 – 11:10 *Coffee break (DFG Lounge MA 315)*

Sunday, December 18th

Chair: Christiane Tretter

11:10 – 11:35 **Yuri Shondin**
Around models of $\mathcal{N}_\kappa^\infty$-functions

11:35 – 12:00 **Henrik Winkler**
Singularities of generalized strings

12:00 – 12:25 **Jussi Behrndt**
Self-adjoint extensions of elliptic partial differential operators on smooth bounded domains

12:25 – 14:25 *Lunch break*

Sunday, December 18th

Chair: Matthias Langer

14:25 – 14:50	**Franciszek H. Szafraniec** A look at the Krein space: New thoughts and old truths
14:50 – 15:15	**Vladimir Strauss** On a model description for normal operators of D_κ^+-class in Krein Spaces
15:15 – 15:40	**Gerald Wanjala** Absolutely p-summing operators in Krein spaces
15:40 – 16:10	*Coffee Break (DFG Lounge MA 315)*

Sunday, December 18th

Chair: Leiba Rodman

16:10 – 16:35	**Christian Mehl** Polar decompositions in indefinite inner product spaces
16:35 – 17:00	**Konstantin Pankrashkin** Cantor spectra on periodic quantum graphs with magnetic fields
17:00 – 17:25	**Peter Jonas** On perturbations of linear relations in Krein spaces
17:25 – 17:30	*Closing*

Some Class of Solvable Potentials for Dirac Type Systems

V. Adamyan

Let $\omega_{\pm}(\lambda)$, $-\infty<\lambda<\infty$, be a function of limited variation such that

$$\sigma(\lambda):=\frac{\lambda}{2\pi}+\omega_{+}(\lambda),$$

is non-decreasing and $\omega_{-}(\lambda)$ is a non-decreasing step function having $\kappa<\infty$ jump discontinuities. Set

$$\begin{aligned} H(t) &= H_{+}(t)-H_{-}(t), \\ H_{\pm}(t) &= \int_{-\infty}^{\infty} e^{-i\lambda t} d\left[\omega_{+}(\lambda)-\omega_{-}(\lambda)\right], \ -\infty<t<\infty. \end{aligned}$$

Then for any positive $r<\infty$ the integral operator $\widehat{H}_r=\widehat{H}_{+,r}-\widehat{H}_{-,r}$ in $\mathbb{L}^2(0,r)$ with the difference kernel $H(t-s)$ is nuclear and there is $r_0>0$ such that for $r\geq r_0$ the operator $I+\widehat{H}_r$ is invertible and has exactly κ negative eigenvalues. For any $r>0$ such that $I+\widehat{H}_r$ is invertible let $\Gamma_r(t,s), 0\leq t,s\leq r$, $r\geq r_0$, be the unique continuous solution of the integral equation

$$\Gamma_r(t,s)+\int_0^r H(t-u)\Gamma_r(u,s)du=H(t-s).$$

We consider the introduced the continual analog of the system of orthogonal trigonometric polynomials on the unit circle introduced by M.G. Krein:

$$e(r,\lambda):=e^{i\lambda r}\left(1-\int_0^r \Gamma_r(s,r)e^{-i\lambda s}ds\right), \ 0\leq r<\infty,$$

$$\left(\int_{-\infty}^{\infty} e(r,\lambda)^* e(r',\lambda)d\left[\sigma(\lambda)-\omega_{-}(\lambda)\right]=0, \ r\neq r', \ r_0<r,r'<\infty\right).$$

Put

$$\begin{aligned} u(r) &= \mathrm{Re}\Gamma_r(0,r), \\ v(r) &= \mathrm{Im}\Gamma_r(0,r), \ r\geq r_0; \\ \varphi(r,\lambda) &:= \mathrm{Re}\left(e^{-i\frac{1}{2}\lambda r}e(r,\lambda)\right), \\ \psi(r,\lambda) &:= \mathrm{Im}\left(e^{-i\frac{1}{2}\lambda r}e(r,\lambda)\right), \ \ \mathrm{Im}\lambda=0. \end{aligned}$$

Then $\varphi(r,\lambda)$, $\psi(r,\lambda)$ satisfy the Dirac type system

$$\begin{aligned} \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} \psi' \\ \varphi' \end{pmatrix} &= \frac{\lambda}{2}\begin{pmatrix} \psi \\ \varphi \end{pmatrix}+V(r)\begin{pmatrix} \psi \\ \varphi \end{pmatrix}, \\ V(r) &= \begin{pmatrix} v(r) & u(r) \\ u(r) & -v(r) \end{pmatrix}. \end{aligned} \tag{1}$$

Let $V_0(r)$ be the potential of the system (1) obtained in this way for the non-decreasing function $\sigma(\lambda)$. Considering $V_0(r)$ and $\sigma(\lambda)$ as known we describe explicitly the amendment $V(r)-V_0(r)$ caused by taking $\omega_{-}(\lambda)$ from $\sigma(\lambda)$.

Discrete Analogs of Canonical Systems with Pseudo–Exponential Potential. Definitions and Formulas for the Spectral Matrix Functions

D. Alpay

joint work with I. Gohberg

We define and study the discrete analogue of canonical differential expressions. We focus on the rational case. Two kinds of discrete systems are to be distinguished: one–sided and two–sided. In both cases the analogue of the potential is a sequence of numbers in the open unit disk (Schur coefficients). We define the characteristic spectral functions of the discrete systems and provide exact realization formulas for them when the Schur coefficients are of a special form called strictly pseudo–exponential. Corresponding inverse problems will also be reviewed.

Non-Self-Adjoint Jacobi Matrices with Rank One Imaginary Part

Y. Arlinskii

joint work with E. Tsekanovskiĭ

We study the spectral problems for finite and semi-finite Jacobi matrices with rank one imaginary part. The non-self-adjoint analogs of the Stone, Hochstadt, and Gesztesy-Simon theorems are established.

On a Generalization of a Pontryagin Space Property

T.Ya. Azizov

Let $\mathcal{K}$ be a Pontryagin space with κ negative squares, let $\mathcal{D} \subset \mathcal{K}$ be a dense linear set. Then $\mathcal{D}$ contains a κ dimensional negative subspace (L.S. Pontryagin). This proposition is very important when one proves invariant subspace theorems in Pontryagin spaces. If $\mathcal{K}$ is a Krein space there is no such a proposition and one has to suppose that the domain of an operator contains a maximal uniformly positive subspace (H. Langer). In the last years A.A. Shkalikov introduced some assumptions for dissipative operators in a Krein space which are sufficient for the existence of maximal semi-definite invariant subspaces and they are more general than the Langer condition.

The aim of this talk is to show that in a natural situation the Shkalikov assumptions imply the Langer condition.

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Self-Adjoint Extensions of Elliptic Partial Differential Operators on Smooth Bounded Domains

J. Behrndt

joint work with M. Langer

It is well-known that all self-adjoint realizations of a regular or singular Sturm-Liouville differential expression can be described with the help of boundary conditions on the functions and their derivatives from the domain of the maximal operator. Moreover the spectral properties of these self-adjoint realizations can be characterized with the help of the Titchmarsh-Weyl function.

The situation is less clear for elliptic partial differential operators. In this talk we consider elliptic second order differential operators on a smooth bounded domain Ω and we discuss which self-adjoint extensions can be described with the help of boundary conditions on $\partial\Omega$. The main difficulty here is, that in contrast to ordinary differential operators the functions in the domain of the maximal operator in general do not assume boundary values on $\partial\Omega$.

We propose a suitable generalization of the boundary triple concept and apply it to elliptic differential operators defined on certain subsets of the domain of the maximal operator. With the help of the associated Weyl function the spectral properties of a class of self-adjoint extensions defined by boundary conditions on $\partial\Omega$ can be characterized.

On the Uniform Convergence of Pade Approximants for Generalized Nevanlinna Functions

M. Derevyagin

joint work with V.A. Derkach

Let φ be given as follows

$$\varphi(\lambda) = r_1(\lambda)\int_a^b \frac{d\mu(t)}{t-\lambda} + r_2(\lambda),$$

where r_1, r_2 are real rational functions such that $r_1(x) \geq 0$ for $x \in \mathbb{R}$, $r_1(x) = O(1)$ as $x \to +\infty$, and $r_2(x) = o(1)$ as $x \to +\infty$. Then *the n-th diagonal Pade approximant for φ* is defined as the rational function $\pi_n(\lambda) = Q_n(\lambda)/P_n(\lambda)$ satisfying the relations

$$\varphi(\lambda) - \pi_n(\lambda) = O(\lambda^{-2n-1}) \ (|\lambda| \to +\infty),$$

$\deg Q_n \leq n$, and $\deg P_n = n$. According to the Pade theorem, there exist n-th diagonal Pade approximants for sufficiently large n. It is proved that *the sequence $\{\pi_n\}_{n=1}^{\infty}$ converges to φ locally uniformly in $\mathbb{C} \setminus ([a,b] \cup \mathcal{P}(\varphi))$*; here $\mathcal{P}(\varphi)$ denotes the set of all poles of φ. A similar statement for a large class of generalized Nevanlinna functions is proven. The main tool of the proof is the generalized Jacobi matrix associated to φ, which corresponds to the Schur algorithm for continued fraction expansion of φ.

On the Schur Transformation of Generalized Nevanlinna Functions

A. Dijksma

joint work with D. Alpay and H. Langer

The classical Schur transformation at $z = 0$,

$$s(z) \longrightarrow s_1(z) = \frac{1}{z}\frac{s(z) - s(0)}{1 - s(0)^* s(z)},$$

maps a Schur function $s(z)$ (holomorphic on the open unit disk and bounded by 1 there) to a Schur function $s_1(z)$. It has been generalized to Schur functions $s(z)$ with finitely many negative squares which are holomorphic at $z = 0$ by C. Chamfy and others. Together with T. Azizov (Voronezh) and G. Wanjala (Mbarara) we have studied the effect of this transformation on the isometric realization of generalized Schur functions and related topics such as the basic interpolation problem, augmented Schur parameters, and factorization of a class of 2×2 matrix polynomials.

In this lecture we extend the Schur transform to Nevanlinna functions with finitely many negative squares which are holomorphic at a fixed point in the open upper half plane.

A Necessary Aspect of the Generalized Beals Condition for the Riesz Basis Property of Indefinite Sturm-Liouville Problems

A. Fleige

For the Sturm-Liouville eigenvalue problem $-f'' = \lambda r f$ on $[-1, 1]$ with Dirichlet boundary conditions and with an indefinite weight function r changing its sign at 0 we discuss the question whether the eigenfunctions form a Riesz basis of the Hilbert space $L^2_{|r|}[-1, 1]$. In the nineties the sufficient so called generalized one hand Beals condition was found for this Riesz basis property. Now using a new criterion of Parfyonov we show that already the old approach gives rise to a necessary and sufficient condition for the Riesz basis property under certain additional assumptions.

On Krein Spaces Induced by Symmetric Operators

A. Gheondea

joint work with P. Cojuhari

We introduce the notion of Krein spaces induced by symmetric densely defined operators in a Hilbert space. We study the existence, relevant examples, uniqueness, and lifting problems. These generalize previous results obtained on the bounded indefinite case and unbounded nonnegative case. The problem is motivated by applications to boundary value problems associated to certain partial differential equations.

Inverse Spectral Problems for Bessel Operators

R. Hryniv

joint work with S. Albeverio and Ya. Mykytyuk

We study the direct and inverse spectral problems for Bessel operators on $(0,1)$ given by the differential expression

$$Sy(x) = -y''(x) + \frac{l(l+1)}{x^2}y(x) + q(x)y(x),$$

and subject to suitable boundary conditions at the point $x = 1$. Here $l \in \mathbb{Z}_+$ is an *angular momentum* and $q \in W_2^{-1}(0,1)$ is a distributional potential. We give a complete description of possible spectra for such operators and solve the inverse problem of reconstructing l and q from the spectral data (two spectra or one spectrum and the corresponding norming constants).

Minimum-Phase Infinite-Dimensional Second-Order Systems

B. Jacob

joint work with K. Morris and C. Trunk

We study second-order systems of the form

$$\ddot{z}(t) + A_o z(t) + D\dot{z}(t) = B_o u(t),$$

equipped with the position measurements

$$y(t) = B_o^* z(t).$$

Here the stiffness operator A_o is a self-adjoint, positive definite, invertible linear operator on a Hilbert space $\mathcal{H}$ and the control operator B_o is a bounded operator acting from $\mathbb{C}^k$ to $\mathcal{H}_{-\frac{1}{2}}$, where $\mathcal{H}_\alpha$, $\alpha \in \mathbb{R}$, is the scale of Hilbert spaces with respect to A_o. Moreover the damping operator $D : \mathcal{H}_{\frac{1}{2}} \to \mathcal{H}_{-\frac{1}{2}}$ is a bounded operator such that $A_o^{-1/2} D A_o^{-1/2}$ is a non-negative operator in $\mathcal{H}$.

The transfer function $G(s) = B_0^*(s^2 I + sD + A_o)^{-1} B_o$ describes the behaviour of the system in the frequency domain. In the case $\mathcal{H}$ equals $\mathbb{C}^n$ a system is called *minimum-phase* if its transfer function G is well-defined and has no zeros in the right-half-plane. A more general definition of minimum-phase systems exists for infinite-dimensional systems.

In order to show that — under some additional assumptions on the damping operator D — these systems are well-posed and minimum-phase, we study the block operator matrix $A = \left[\begin{smallmatrix} 0 & I \\ -A_o & -D \end{smallmatrix}\right]$. As an example an Euler-Bernoulli beam with Kelvin-Voigt damping is considered.

On Perturbations of Linear Relations in Krein Spaces

P. Jonas

joint work with T. Azizov, J. Behrndt and C. Trunk

In this talk we study the behaviour of spectral points of positive type and of type π_+ of linear relations under compact perturbations and perturbations of small size.

On Eigenvalues of Non-Definitizable Differential Operators

I. Karabash

Let $L_{sl} = -\frac{d^2}{dx^2} + q(x)$ be a selfadjoint Sturm-Liouville operator in $L^2(\mathbb{R})$. Let $(Jf)(x) := (\operatorname{sgn} x) f(x)$. The problem under consideration is a detailed description of the spectrum of the J-selfadjoint Sturm-Liouville operator

$$A_{sl} := JL_{sl} = (\operatorname{sgn} x)\left(-\frac{d^2}{dx^2} + q(x)\right).$$

The theory of boundary triples (see, for example, [1]) gives a description of the spectrum $\sigma(A_{sl})$ and the discrete spectrum $\sigma_{disc}(A_{sl})$ in terms of the Weyl function. In this work we consider the question of the existence of eigenvalues in the essential spectrum $\sigma_{ess}(A_{sl})$ of the singular J-selfadjoint Sturm-Liouville operator A_{sl}. Geometric and algebraic multiplicities of eigenvalues will be given. Emphasize that we do not assume that the operator A_{sl} is definitizable.

We apply these results to obtain the following theorems.

Theorem 1 Let $L_{sl} = -\frac{d^2}{dx^2} + q$ be a Sturm-Liouville operator with a finite-zone potential. Then

(i) the nonreal spectrum of A_{sl} consists of a finite number of eigenvalues;

(ii) eigenvalues of A_{sl} are isolated and have finite algebraic multiplicity.

Theorem 2 There exist a selfadjoint Krein-Feller differential operator $L = -\frac{d^2 f}{dM(x)dx}$ in $L^2(\mathbb{R}, dM(x))$ and a positive constant c such that the operator

$$A = J(L - c) = (\operatorname{sgn} x)\left(-\frac{d^2 f}{dM(x)dx} - c\right)$$

has an eigenvalue of infinite algebraic multiplicity.

[1] V. A. Derkach, M. M. Malamud, *The extension theory of Hermitian operators and the moment problem*, Analiz-3, Itogi nauki i tehn. Ser. Sovrem. mat. i eё pril., V. 5, VINITI, Moscow, 1993 (Russian); English translation: J. Math. Sc.—1995.—V. 73, no. 2.—P.141–242.

An Analogue of the Liouville Theorem for Linear Relations in Banach Spaces

V. Khatskevich

joint work with M.I. Ostrovskii and V.S. Shulman

Consider a bounded linear operator T between Banach spaces $\mathcal{B}$, $\mathcal{B}'$ which can be decomposed into direct sums $\mathcal{B} = \mathcal{B}_1 \oplus \mathcal{B}_2$, $\mathcal{B}' = \mathcal{B}'_1 \oplus \mathcal{B}'_2$. Such linear operator can be represented by a 2×2 operator matrix of the form

$$T = \begin{pmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{pmatrix},$$

where $T_{ij} \in \mathcal{L}(\mathcal{B}_j, \mathcal{B}'_i)$, $i, j = 1, 2$. (By $\mathcal{L}(\mathcal{B}_j, \mathcal{B}'_i)$ we denote the space of bounded linear operators acting from $\mathcal{B}_j$ to $\mathcal{B}'_i$ $(i, j = 1, 2)$.) The map G_T from $\mathcal{L}(\mathcal{B}_1, \mathcal{B}_2)$ into the set of closed affine subspaces of $\mathcal{L}(\mathcal{B}'_1, \mathcal{B}'_2)$, defined by

$$G_T(X) = \{Y \in \mathcal{L}(\mathcal{B}'_1, \mathcal{B}'_2) : \ T_{12} + T_{22}X = Y(T_{11} + T_{12}X)\}$$

is called a *linear fractional relation* (LFR) (associated with T).

Such relations can be considered as a generalization of linear fractional transformations which were studied by many authors and found many applications. In this talk we continue the study of LFR, the talk is devoted mostly to analogous of the Liouville theorem "a bounded entire function is constant" for LFR.

Spectral Estimates for Schrödinger Operators with Periodic Matrix Potentials on the Real Line

E. Korotyaev

We consider the Schrödinger operator on the real line with a $N \times N$ matrix valued periodic potential, $N > 1$. The spectrum of this operator is absolutely continuous and consists of intervals separated by gaps. We define the Lyapunov function, which is analytic on an associated N-sheeted Riemann surface. On each sheet the Lyapunov function has the standard properties of the Lyapunov function for the scalar case. The Lyapunov function has (real or complex) branch points, which we call resonances. We determine the asymptotics of the periodic, anti-periodic spectrum and of the resonances at high energy (in terms of the Fourier coefficients of the potential). We show that there exist two types of gaps:

i) stable gaps, i.e., the endpoints are periodic and anti-periodic eigenvalues,

ii) unstable (resonance) gaps, i.e., the endpoints are resonances (real branch points).

Moreover, the following results are obtained:

1) we define the quasimomentum as an analytic function on the Riemann surface of the Lyapunov function; various properties and estimates of the quasimomentum are obtained,

2) we construct the conformal mapping with real part given by the integrated density of states and imaginary part given by the Lyapunov exponent. We obtain various properties of this conformal mapping, which are similar to the case N=1,

3) we determine various new trace formulae for potentials and the Lyapunov exponent,

4) a priori estimates of gap lengths in terms of potentials are obtained.

Similarity of some Indefinite Sturm-Liouville Operators to Self-Adjoint Operators

A. Kostenko

Let $\omega(\cdot) : \mathbb{R} \to \mathbb{R}_+$ be a positive measurable function on $\mathbb{R}$ and let $L^2(\mathbb{R},\ \omega)$ stand for the weighted Hilbert space consisting of measurable functions f on $\mathbb{R}$ that satisfy $\int_{\mathbb{R}} |f(t)|^2 \omega(t)dt < \infty$.

(i) Consider an operator

$$L_c := \frac{\operatorname{sgn} x}{|x|^\alpha}\left(-\frac{d^2}{dx^2} + c\delta\right), \quad c \in \mathbb{C}, \qquad \alpha > -1,$$

acting in $L^2(\mathbb{R},\ |x|^\alpha)$. Here δ is the Dirac delta. It is called an operator with a point interaction at zero.

We find a criterion for the operator L_c to be similar to a normal (self-adjoint) operator in terms of the interaction c.

(ii) Let $\omega(x) = p(x)|x|^\alpha, (\alpha > -1)$, and $0 < d \le p(x) \le D < \infty$. We find some sufficient conditions for an J–selfadjoint operator

$$L_\omega := -\frac{\operatorname{sgn} x}{p(x)|x|^\alpha}\frac{d^2}{dx},$$

in $L^2(\mathbb{R},\ \omega)$ to be similar to a selfadjoint operator. These conditions are expressed in terms of the function $p(\cdot)$.

The method of investigation of critical points of a definitizable operator of such a type is based on the Naboko–Malamud resolvent similarity criterion. Moreover, we exploit the theory of boundary triplets of symmetric operators and the corresponding Weyl functions.

Sturm-Liouville Operators with Singularities and Generalized Nevanlinna Functions

H. Langer

Following C. Fulton, for a class of Sturm-Liouville operators with a singularity at zero (which contains e.g. the Bessel operator) a special Titchmarsh–Weyl coefficient $m(z)$ is defined, and it is shown that $m(z)$ belongs to some generalized Nevanlinna class.

Variational Principles for Eigenvalues of the Klein–Gordon Equation

M. Langer

joint work with C. Tretter

In this talk we consider eigenvalues of the Klein–Gordon equation, which can be written as a quadratic eigenvalue problem. Under certain assumptions the continuous spectrum has a gap and we can characterise eigenvalues in this gap even in the presence of complex eigenvalues. As a consequence we can compare these eigenvalues with eigenvalues of certain Schrödinger operators.

On the Eigenvalues of Self-Adjoint Extensions with Exit

A. Luger

joint work with J. Behrndt

Let A be a symmetric operator in a Krein space $\mathcal{K}$ and suppose that it has a self-adjoint extension A_0 in $\mathcal{K}$ that locally in some domain Ω behaves like a self-adjoint relation in a Pontryagin space (that is, A is of type π_+ over Ω). Then we are interested in the eigenvalues of self-adjoint extensions $\widetilde{A}$ in a larger space $\widetilde{\mathcal{K}} \supset \mathcal{K}$, under the assumption that $\widetilde{A}$ also is of type π_+ over Ω.

We are giving an analytic characterization in terms of the functions $m(\lambda)$, which is the Q- (or Weyl) function of the pair (A, A_0), and the parameter $\tau(\lambda)$ in

$$P_{\mathcal{K}}\big(\widetilde{A}-\lambda\big)^{-1}|_{\mathcal{K}} = (A_0-\lambda)^{-1} - \gamma(\lambda)\big(m(\lambda)+\tau(\lambda)\big)^{-1}\gamma(\overline{\lambda})^{+},$$

where $\gamma(\lambda)$ denotes the corresponding γ-field.

In particular, it follows that $\widetilde{A}$ in general is not a minimal representing relation for the function $-\frac{1}{m(\lambda)+\tau(\lambda)}$ as it is in the case of canonical extensions, where τ is a constant. Furthermore, we give a necessary condition on the spectral measure of A_0 for the existence of embedded eigenvalues for some extension $\widetilde{A}$.

Weyl Functions and Generalization of the Aronszajn-Donoghue Theory

M. Malamud

joint work with H. Neidhardt

We investigate the singular spectrum of selfadjoint extensions of a symmetric operator with equal deficiency indices. The technique of boundary triplets and the corresponding Weyl functions is used.

Polar Decompositions in Indefinite Inner Product Spaces

C. Mehl

joint work with B. Lins, P. Meade, A. Ran and L. Rodman

The talk gives an overview of the theory of polar decompositions in a space equipped with an indefinite inner product, i.e., decompositions of matrices into two factors that are unitary and selfadjoint with respect to this indefinite inner product. The talk discusses basic properties as well as recently obtained results. The main focus are finite dimensional spaces, but also generalizations to Pontryagin and Krein spaces are briefly discussed.

A Completeness Theorem for a Non-Standard Two-Parameter Eigenvalue Problem

M. Möller

joint work with M. Faierman and B. Watson

We consider two simultaneous Sturm-Liouville systems involving two spectral parameters,

$$\lambda y''(s) + \mu a(s)y(s) + b(s)y(s) = 0, \ 0 \le s \le 1, \ ' = d/ds, \quad (1)$$
$$y(0) = y(1) = 0, \quad (2)$$
$$-\lambda c(t)z(t) + \mu z''(t) + d(t)z(t) = 0, \ 0 \le t \le 1, \ ' = d/dt, \quad (3)$$
$$z(0) = z(1) = 0, \quad (4)$$

where a, b, c, and d are real-valued and continuous, and in addition, a and c are positive, and at least one of the functions b and d has no zeros, and if $b \neq 0$ (resp. $d \neq 0$), then b (resp. d) is not a scalar multiple of a (resp. c).

Eliminating the eigenvalue parameter μ, the boundary problem

$$\lambda(D_1^2 D_2^2 u + acu) + (bD_2^2 u - adu) = 0 \quad \text{in} \quad \Omega, \quad (5)$$
$$u = 0 \quad \text{on} \quad \Gamma, \quad (6)$$

arises, where $D_1 = \partial/\partial s$, $D_2 = \partial/\partial t$, $\Omega = (0,1) \times (0,1)$, and $\Gamma = \partial\Omega$.

Since the differential equation (5) is not elliptic, the usual methods used in standard multiparameter spectral theory to obtain the required regularity results for solutions of (5)-(6) no longer apply. However, we are able to show that the domain of the self-adjoint operator associated with (5)-(6) contains the Sobolev space $H^2(\Omega)$, which enables us to show that the eigenvectors of (1)-(4) are complete in $L^2(\Omega)$ in case $b = 0$ or $d = 0$.

In case $b \neq 0$, the operator A has essential spectrum in $\mathbb{R}\backslash\{0\}$, and the completeness of the eigenvectors is still an open problem.

Block Matrices, Boundary Triplets and Scattering

H. Neidhardt

joint work with J. Behrndt, M.M. Malamud and J. Rehberg

For an operator-valued 2×2 block-matrix, which is called a Feshbach decomposition in physics, a scattering theory is considered. Under trace class perturbation the channel scattering matrices are calculated. Using Feshbach's optical potential it is shown that for a given spectral parameter the channel scattering matrices can be recovered either from a dissipative or from a Lax-Phillips scattering theory.

The results are extended to singular perturbations, that is, to scattering systems $\{L, A_0 \oplus T_0\}$ where A_0 and T_0 are self-adjoint extensions of symmetric operators A and T, respectively, and L is an appropriate self-adjoint extension of the symmetric operator $A \oplus T$. It turns out that the notion of optical potentials corresponds to the notion of Strauss family.

Cantor Spectra on Periodic Quantum Graphs with Magnetic Fields

K. Pankrashkin

joint work with J. Brüning and V. Geyler

For a class of square planar lattices placed into a magnetic field we reduce the spectral problem to the discrete magnetic Laplacian using self-adjoint extensions and Krein's resolvent formula. The recently solved ten martini problem and the well-known correspondence between the discrete magnetic Laplacian and the almost Mathieu operator imply that the spectrum is a Cantor set for the case of an irrational flux quanta number through the elementary cell of the lattice.

Spectral Points of Type π_+ of Symmetric Operators in Indefinite Inner Product Spaces

F. Philipp

joint work with C. Trunk

Let G be a bounded self-adjoint operator in a Hilbert space $(\mathcal{H}, (\cdot,\cdot))$. We define an inner product by $[\cdot,\cdot] = (G\cdot,\cdot)$. Let A be a closed operator in $\mathcal{H}$ satisfying the symmetry condition

$$[Ax, y] = [x, Ay] \text{ for all } x, y \in \mathcal{D}(A).$$

We call such an operator *G-symmetric*. An approximative eigenvalue $\lambda \in \sigma_{ap}(A)$ is called a *spectral point of type π_+ of A*, if there exists a linear manifold $\mathcal{H}_\lambda$ in $\mathcal{H}$ with finite codimension, such that for every approximative eigensequence (x_n) (i.e. $\|x_n\| = 1$ and $(A-\lambda)x_n \to 0$ as $n \to \infty$) in $\mathcal{H}_\lambda \cap \mathcal{D}(A)$ the accumulation points of the sequence $([x_n, x_n])$ are positive. We denote the set of all those spectral points by $\sigma_{\pi_+}(A)$.

For $\lambda \in \sigma_{\pi_+}(A)$ we describe all the possible linear manifolds $\mathcal{H}_\lambda$ with the property mentioned above and we determine among them a special manifold $\mathcal{H}_\lambda$ with minimal codimension. Let A_1 be another G-symmetric operator such that $\rho(A) \cap \rho(A_1) \neq \emptyset$ and $(A-\mu)^{-1} - (A_1-\mu)^{-1}$ is compact. Then we show that λ either is contained in $\sigma_{\pi_+}(A_1)$ or in $\mathbb{C} \setminus \sigma_{ap}(A_1)$.

For real intervals $[a,b]$ with $[a,b] \cap \sigma_{ap}(A) \subset \sigma_{\pi_+}(A)$ we prove, that there is an open neighbourhood U of $[a,b]$ in $\mathbb{C}$, such that either all points in U are eigenvalues of A or $(U \setminus \mathbb{R}) \cap \sigma_{ap}(A)$ is empty. If even $U \setminus \mathbb{R} \subset \rho(A)$ holds, then the norm of the resolvent $(A-\mu)^{-1}$ can be estimated by some power of $|\mathrm{Im}\,\mu|^{-1}$ in a neighbourhood of $[a,b]$.

Moreover, we show the existence of a local spectral function E of A on (a,b). The spectral subspace $E(\Delta)\mathcal{H}$ is always the direct G-orthogonal sum of a Pontryagin space and a finite dimensional neutral subspace.

Shifted Hermite-Biehler functions.
II. Applications

V. Pivovarchik

joint work with H. Woracek

The general result presented in the lecture by H. Woracek devoted to the order in location of pure imaginary zeros of functions from the class $\mathcal{T}(HB^{sb}_{<\infty})$ is applied to describe eigenvalues of the following boundary value problems: the original and generalized Regge problems, a problem describing small transversal vibrations of a damped string, a problem describing small transversal vibrations of a damped elastic compressed rod. These problems are generated by ordinary differential equations and boundary conditions depending on the spectral parameter. The order in location of pure imaginary eigenvalues can be used for solving the corresponding inverse problems, i.e. problems of recovering the coefficients of equations and the boundary conditions using the spectrum of the boundary value problem.

Another approach to the mentioned boundary value problems is connected with the theory of quadratic operator pencils.

Some Properties of Eigenfunctions and Associated Functions of Indefinite Sturm-Liouville Problems

S.G. Pyatkov

We consider the problem

$$Lu = \lambda g(x)u, \quad x \in (a,b), \tag{1}$$

where L is an ordinary differential operator of order $2m$ which is defined by the differential expression

$$Lu = \sum_{i,j=1}^{m} \frac{d^i}{dx^i} a_{ik} \frac{d^j u}{dx^j} \quad (x \in (a,b)) \tag{2}$$

and the boundary conditions

$$B_k u = \sum_{i=0}^{2m-1} (\alpha_{ik} u^{(i)}(a) + \beta_{ik} u^{(i)}(b)) = 0 \ (k = 1,2,\dots,2m)$$

or

$$B_k u = \sum_{i=0}^{2m-1} \alpha_{ik} u^{(i)}(a) = 0, \quad (k = 1,2,\dots,m),$$
$$\lim_{x\to+\infty} u^{(i)} = 0 \ (i = 0,1\dots m-1),$$

or

$$B_k u = \sum_{i=0}^{2m-1} \beta_{ik} u^{(i)}(b) = 0 \ (k = 1,2,\dots,m),$$
$$\lim_{x\to-\infty} u^{(i)} = 0 \quad (i = 0,1\dots m-1),$$

or

$$\lim_{x\to\infty} u^{(i)} = 0 \quad (i = 0,1\dots m-1).$$

Here the first conditions are used in the case of a bounded interval (a,b), the second in the case of $b = +\infty$, the third when $a = -\infty$, and the last conditions in the case of $(a,b) = \mathbb{R}$. The operator L is assumed to be selfadjoint in $L_2(a,b)$.

We study the Riesz basis property of the root functions of the problem (1) in the weighted space $L_{2,g}((a,b)\setminus G^0)$ ($G^0 = \{x \in (a,b) : g(x) = 0\}$) endowed with the norm

$$\|u\|^2_{L_{2,g}((a,b)\setminus G^0)} = \int\limits_{(a,b)\setminus G^0} |g||u|^2(x)\ dx.$$

Assume that there exist open subsets G^+ and G^- of $G = (a,b)$ such that $\mu(\overline{G^\pm}\setminus G^\pm) = 0$, $g(x) > 0$ a.e. (almost everywhere) in G^+, $g(x) < 0$ a.e. in G^-, and $g(x) = 0$ a.e. in $G^0 = G\setminus\overline{G^+}\cup\overline{G^-}$). Here μ is the Lebesgue measure. A point $x_0 \in \partial G^+ \cap \partial G^-$ is called a turning point. We show that in almost all cases the Riesz basis property is independent of the boundary conditions. Next, we demonstrate that the Riesz basis property depends only on the behavior of the function g at the turning points and thus it does not matter how this function looks like beyond some neighborhood about the set of turning points. At last, with the use of new sufficient conditions of regularity of a turning point obtained by Parfenov A.I. we refine some previous results and present some sufficient conditions of the Riesz basis property.

Semidefinite Invariant Subspaces for Normal and Hyponormal Matrices in an Indefinite Inner Product Space

A. Ran

joint work with C. Mehl and L. Rodman

We discuss extension of a given semidefinite invariant subspace for a normal matrix to a maximal semidefinite invariant subspace. Next, generalizations to hyponormal matrices are discussed.

Canonical Forms for Pairs of Symmetric, Skew Symmetric, or Hermitian Matrices: Some Applications and Open Problems

L. Rodman

The canonical forms for pairs of real or complex symmetric, skew symmetric, or Hermitian matrices are well known. In recent years, there has been a recurring interest in these forms. Some new applications of the forms will be given, in particular, concerning geometry of joint numerical ranges. Open problems will be formulated in which the canonical forms are likely to play a role.

Extension Theory of Sectorial Linear Relations. A Factorization Approach

A. Sandovici

joint work with S. Hassi, H. de Snoo and H. Winkler

The Kreĭn-von Neumann and the Friedrichs extensions of a sectorial relation are characterized in terms of factorizations. For this purpose, the one-to-one correspondence between densely defined closed sectorial forms and maximal sectorial operators is extended to the case of nondensely defined closed sectorial forms and maximal sectorial linear relations, respectively. As in the case of nonnegative linear relations, these factorizations lead to a novel approach to the transversality and equality of the Kreĭn-von Neumann and the Friedrichs extensions and to the notion of positive closability (the Kreĭn-von Neumann extension being an operator). Furthermore, all extremal extensions of the linear relation are characterized in terms of analogous factorizations. This approach for the general case of sectorial linear relations in a Hilbert space extends the applicability of such factorizations. In particular, all maximal sectorial extremal extensions of a bounded sectorial linear operator are characterized.

PT- and CPT-Symmetric Operators. Critical Parameter Values. Spectral Portraits.

A. Shkalikov

joint work with S. Albeverio and P. Kurasov

The concept of PT- and CPT- symmetric operators appeared recently in physical literature. A typical example is an operator of the form

$$-\varepsilon d/dx^2 + u(x) + iv(x)$$

with Dirichlet boundary conditions at the interval (-1,1), where $u(x)$ and $v(x)$ are real even and odd functions, respectively, and ε is (the so-called quasi-classical) parameter. We shall discuss the spectral properties of this operator family (and other ones).

Around Models of $\mathcal{N}_\kappa^\infty$–Functions

Y. Shondin

joint work with A. Dijksma and A. Luger

Each generalized Nevanlinna functions $N(z) \in \mathcal{N}_\kappa$ for which $z = \infty$ is the only pole (or generalized pole) of non-positive type is characterized by the *irreducible* representation

$$N(z) = (z - z_0)^m (z - z_0^*)^m (N_0(z) + q(z)) + p(z), \qquad (1)$$

where z_0 belongs to the domain of holomorphy of N, $m \in \mathbb{N}_0$, $N_0(z) \in \mathcal{N}_0$, $q(z)$ and $p(z)$ are real polynomials, $\deg p \leq 2m-1$. It can be shown that $N(z)$ in (1) admits also another representation

$$N(z) = (z - z_0)^\kappa (z - z_0^*)^\kappa N_0^r(z) + r(z), \qquad (2)$$

with some $N_0^r(z) \in \mathcal{N}_0$ and a real polynomial $r(z)$ of degree $\leq 2\kappa$–1. In general the representation (2) is *reducible*. Earlier we have described minimal representations of $N(z)$ and $-N(z)^{-1}$ in the reproducing kernel space $\mathcal{L}(N_0) \oplus \mathcal{L}(q) \oplus \mathcal{L}(M_p)$ associated with the factors N_0, q and $((z - z_0)^m, p)$ of the irreducible representation (1). Here we consider models of $N(z)$ and $-N(z)^{-1}$ in the reproducing kernel space $\mathcal{L}(N_0^r) \oplus \mathcal{L}(M_r)$ associated with the factors N_0^r and $((z - z_0)^\kappa, r)$ of the representation (2) and also a correspondence between the models. The meaning of representation (2) and associated models for an approximation problem will be discussed.

Limit-Point/Limit-Circle Classification for Sturm-Liouville Problems whose Coefficients Depend Rationally on the Eigenvalue Parameter

H. de Snoo

joint work with S. Hassi and M. Möller

Weyl's limit-point/limit-circle alternative states that for every non-real λ the set of solutions of $(-DpD + q)y = \lambda y$ (the function q is real and measurable, and D denotes differentiation with respect to the single variable) belonging to $L^2(0, \infty)$ is a vector space of dimension 1 or 2, and secondly that either for each $\lambda \in \mathbb{C}$ this solution space has dimension 2 or for each $\lambda \in \mathbb{C}$ its dimension is at most 1. In this talk similar statements are considered for the 2×2 system

$$\mathbb{A}_0 = \begin{pmatrix} -DpD + q & -Dc + a \\ cD + a & r \end{pmatrix} \qquad (7)$$

of formal differential operators, where the coefficient functions $p, q, c, r, a : (0, \infty) \to \mathbb{R}$ are measurable functions with $p \neq 0$ a. e.

On a Model Description for Normal Operators of D_κ^+-Class in Krein Spaces

V. Strauss

In this work a functional model representation of a normal operator N acting in a Krein space is considered. We assume that N and its adjoint operator $N^\#$ have a common invariant subspace L_+ which is a maximal nonnegative subspace and has a representation as a sum of a finite-dimensional neutral subspace and a uniformly positive subspace (i.e. N belongs to the so-called D_κ^+ -class). For N we construct a model representation as the multiplication operator by a scalar function acting in a suitable functional space. This representation is applied to the existence problem for a square root for the operator $N^\# N$ and other problems related to the polar representation for normal operators of D_κ^+-class.

Quadratic Operator Pencils and Selfadjoint Operators in Krein Space

L.I. Sukhocheva

It is well-known that one can study spectral properties of a selfadjoint quadratic pencil via spectral properties of its linearization, which is a selfadjoint operator in a Krein space. We consider the, in some sense, "inverse" problem: to describe the selfadjoint operators in Krein spaces which are linearizations of some selfadjoint quadratic pencil. It is shown that each positive operator has this property.

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A look at the Krein Space: New Thoughts on Old Truths

F.H. Szafraniec

The purpose is twofold: to revive an old idea of Aronszajn and to create a common framework for better understanding of diverse dilation/extention results.

The Spectra of Multiplication Operators Associated with Families of Operators

C. Tretter

joint work with M. Moeller and R. Denk

If the highest derivatives in a system of partial differential equations do not contain derivatives with respect to a certain variable, this variable can be used as a parameter for a corresponding family of operators. We will study the spectrum of multiplication operators associated with such operator families. As an example, the natural oscillations of an incompressible fluid in the neighbourhood of an elliptical flow is considered.

Extension of Symmetric Operators with Finitely Many Negative Squares in Krein Spaces

C. Trunk

joint work with J. Behrndt

Let S be a simple symmetric operator of defect one in a Krein space $\mathcal{K}$ and assume that S has finitely many negative squares and a canonical self-adjoint extension with a nonempty resolvent set. We investigate the number of negative squares of self-adjoint extensions $\widetilde{A}$ of S which act in a larger space $\mathcal{K} \times \mathcal{H}$.

In particular we obtain a characterization of the number of negative squares of the canonical self-adjoint extensions of a simple symmetric operator S of defect one with finitely many negative squares.

The main tool are functions belonging to a so-called class D_κ, which is a subclass of the definitizable functions. We say that a function τ, meromorphic in $\mathbb{C}\backslash\mathbb{R}$, symmetric with respect to $\mathbb{R}$ and holomorphic at λ_0 belongs to the class D_κ, $\kappa \in \mathbb{N}_0$, if there exists a generalized Nevanlinna function $G \in N_\kappa$ holomorphic at λ_0 and a rational function g holomorphic in $\overline{\mathbb{C}}\backslash\{\lambda_0, \overline{\lambda}_0\}$ such that

$$\frac{\lambda}{(\lambda-\lambda_0)(\lambda-\overline{\lambda}_0)}\tau(\lambda) = G(\lambda) + g(\lambda)$$

holds for all points λ where τ, G and g are holomorphic.

New Perturbation Estimates for Eigenvalues and Eigenvectors of Selfadjoint Operators

K. Veselić

We give some new perturbation bounds for the eigenvalues and eigenvectors of (standard) selfadjoint operators, if the perturbation of the operator is measured in 'natural input parameters', e.g. if a Schroedinger operator potential $V(x)$ is perturbed as

$$V(x) \mapsto V(x) + \delta V(x), \quad |\delta V(x)| \leq \varepsilon |V(x)|$$

The possibilities to extend this to indefinite scalar products may well exist, but they do not seem to be straightforward.

Absolutely p-Summing Operators in Krein Spaces

G. Wanjala

Let $\mathcal{H}$ and $\mathcal{K}$ be Krein spaces and $1 \leq p < \infty$. A linear operator $u : \mathcal{H} \to \mathcal{K}$ is *absolutely p-summing* if there is a constant $c > 0$ such that for each positive integer m and any vectors $x_1, \ldots, x_n$ in $\mathcal{H}$ we have

$$\left(\sum_{i=1}^{m} \| u x_i \|^p \right)^{1/p} \leq \sup \left\{ \left(\sum_{i=1}^{m} |\langle y, x_i \rangle|^p \right)^{1/p} : y \in \mathcal{H}, \| y \| \leq 1 \right\}.$$

We show that a linear map $u : \mathcal{H} \to \mathcal{K}$ is absolutely p-summing precisely when it takes weakly p-summable sequences in $\mathcal{H}$ to strongly p-summable sequences in $\mathcal{K}$. We also show that the composition of a p-summing operator with any bounded linear operator is absolutely p-summing. We shall restrict our discussion to the case $p = 2$.

Singularities of Generalized Strings

Henrik Winkler

joint work with M. Kaltenbäck and H. Woracek

A function $q(z)$ belongs to the class N_κ^+ if $q(z) \in N_\kappa$ and $zq(z) \in N$. Each $q(z) \in N_\kappa^+$ is the Titchmarsh - Weyl coefficient of a generalized string, which may have negative jumps or singularities in its mass function, or dipoles. Relations between semibounded and symmetric Pontryagin spaces of entire functions lead to structure results for the spaces which are connected with the critical points of the generalized string, and which can be expressed in terms of the mass and the dipole function. The tool are transformations between maximal chains of matrix functions with $q(z)$, $zq(z)$, and $zq(z^2)$ as Weyl coefficients.

Shifted Hermite-Biehler Functions

Harald Woracek

joint work with V. Pivovarchik

We investigate a certain subclass $\mathcal{S}\!H\!B$ of indefinite Hermite-Biehler functions. Our aim is to characterize the belonging of a function to this subclass by means of the distribution of its zeros. Functions of the class $\mathcal{S}\!H\!B$ appeared in various contexts. For example in the theory of generalized strings, the Regge problem, or the investigation of the vibrations of a damped string, where also the distribution of their zeros, in particular the purely imaginary zeros, proved to be of interest.

Let us describe our result in more detail. If E is an entire function, we say $E \in \mathcal{H}B_{<\infty}$ (E is an *indefinite Hermite-Biehler function*), if $E(z)$ and $E^{\#}(z) := \overline{E(\overline{z})}$ do not have common nonreal zeros and if the kernel

$$S(z,w) := \frac{i}{z-\overline{w}}\Big[1 - \frac{E^{\#}(z)}{E(z)}\overline{\Big(\frac{E^{\#}(w)}{E(w)}\Big)}\Big]$$

has a finite number of negative squares. We write $E \in \mathcal{H}B^{sym}_{<\infty}$ (E is *symmetric*), if in addition to $E \in \mathcal{H}B_{<\infty}$ the functional equation $E^{\#}(z) = E(-z)$ is satisfied. We write $E \in \mathcal{H}B^{sb}_{<\infty}$ (E is *semibounded*), if in addition to $E \in \mathcal{H}B_{<\infty}$ the function $E + E^{\#}$ has only finitely many zeros in $(-\infty, 0)$. The transformation

$$\mathfrak{T} : \big\{E \in \mathcal{H}B^{sb}_{<\infty} : \ E(t) \neq 0, t \in (-\infty,0)\big\} \to \mathcal{H}B^{sym}_{<\infty}$$

defined as $(\mathfrak{T}E)(z) := A(z^2) - izB(z^2)$, where $A := \frac{1}{2}(E + E^{\#})$ and $B := \frac{i}{2}(E - E^{\#})$, is a bijection.

The subclass $\mathcal{S}\!H\!B$ under consideration is now

$$\mathcal{S}\!H\!B := \mathfrak{T}\Bigg(\Bigg\{E \in \mathcal{H}B^{sb}_{<\infty} : \begin{array}{c} E(t) \neq 0, t \in (-\infty,0), \\ S(z,w) \text{ is positive semidefinite} \end{array}\Bigg\}\Bigg)$$

The result we present can be roughly formulated as follows: A function $E \in \mathcal{H}B^{sym}_{<\infty}$ actually belongs to $\mathcal{S}\!H\!B$ if and only if all zeros of F in the upper half plane are simple, lie on the imaginary axis, and if their location governs the distribution of zeros on the negative imaginary axis in a specific way.