Extrema of Branching Processes and Gaussian Free Fields (BPGFF 2014)

WIAS / TU Berlin, November 28 – 29, 2014

Workshop program

Friday, 28. November: **WIAS, Room Erhard-Schmidt Hörsaal**

09:00	*Registration and Refreshments*
09:25	*Opening of the Workshop*
09:30	**Ofer Zeitouni** Extremal processes and freezing
10:30	*Coffee Break*
11:00	**Nicola Kistler** A multiscale refinement of the second moment method
12:00	**Lisa Hartung** Variable speed Branching Brownian motion
13:00	*Lunch (Self Catered)*
14:30	**Elie Aïdékon** On the extremal process of the branching Brownian motion
15:30	*Coffee Break*
16:00	**Rémi Rhodes** Liouville quantum gravity on the Riemann sphere
17:00	**Vincent Vargas** Some new estimates on the Liouville heat kernel
18:00	*Reception*

Saturday, 29. November: **TU, Room MA043**

09:00	**Olivier Zindy** Log-correlated Gaussian fields: study of the Gibbs measure
10:00	**Rajat Subhra Hazra** Thick points for Gaussian free fields with different cut-offs
11:00	*Coffee Break*
11:30	**Oren Louidor** Conformal symmetries in the extremal process of the discrete Gaussian free field
12:30	**Jian Ding** Convergence of the maximum for log-correlated Gaussian fields
13:30	*Conclusion of the Workshop*

Speaker Abstracts

Elie Aïdékon (Université Pierre et Marie Curie, Paris)

On the extremal process of the branching Brownian motion

We give the construction of the extremal process of the branching Brownian motion in terms of a decorated Poisson point process. Joint work with J. Berestycki, E. Brunet and Z. Shi.

Jian Ding (University of Chicago)

Convergence of the maximum for log-correlated Gaussian fields

In this talk, I will show that the re-centered maximum for log-correlated Gaussian field in any dimension converges in law, under the assumption that the covariances of the fields converge in a suitable sense. Our assumption on the covariances is rather minimal and in particular it should imply that the re-centered maximum for the four-dimensional Gaussian membrane model converges in law. This is based on a joint work that is in preparation with Rishideep Roy and Ofer Zeitouni.

Lisa Hartung (Universität Bonn)

Variable speed Branching Brownian motion

Gaussian processes indexed by trees form an interesting class of correlated random fields where the extremal behaviour can be studied. A rich class of examples are "variable speed" Branching Brownian motions which are defined as ordinary BBM with a time-inhomogeneous variance. In this talk I explain how the convergence of the extremal process for variable speed BBMs is obtained, when the "speed functions" describing the time-inhomogeneous variance lie below their concave hull. The resulting limiting objects turn out to be universal in the sense that they only depend on the slope of the speed function at 0 and the final time t. (joint work with A. Bovier)

Rajat Subhra Hazra (Indian Statistical Institute, Kolkata)

Thick points for Gaussian free fields with different cut-offs

Massive and massless Gaussian free fields can be described as generalized Gaussian processes indexed by an appropriate space of functions. In this talk we survey various approaches to approximate these fields and look at the fractal properties of the thick points of their cut-offs. Under some sufficient conditions for a centered Gaussian process with logarithmic diverging variance we study the set of thick points and derive their Hausdorff dimension. We show that various cut-offs for Gaussian free fields satisfy these assumptions. We also give sufficient conditions for comparing thick points of different cut-offs. There are certain approximations which also do not follow some these sufficient conditions, for example, circle average process in two and four dimensions respectively. We also describe the properties of these processes. This is a joint work with Alessandra Cipriani.

Nicola Kistler (Goethe Universität Frankfurt)

A multiscale refinement of the second moment method

I will discuss a refinement of the second moment method which is particularly efficient to analyze the extremes of random fields where multiple scales can be identified. The method emerged from works on the extremes of branching Brownian motion, joint with Louis-Pierre Arguin (CUNY) and Anton Bovier (Bonn), and from a recent work with David Belius (NYU) on the cover time by planar Brownian motion. Time permitting, I will also discuss a procedure of local projections which allows, in a number of models, to generate the scales from first principles.

Oren Louidor (Technion, Haifa)

Conformal symmetries in the extremal process of the discrete Gaussian free field

We study the extremal process associated with the Discrete Gaussian Free Field on the square lattice and show how the conformal symmetries manifest themselves in the scaling limit. Specifically, we prove that the joint process of spatial positions and centered values of the extreme local maxima in lattice versions of a bounded domain D converges, as the lattice spacing tends to zero, to a Poisson point process with intensity given by the product of a random measure Z_D (position) with an exponential measure (value).

The laws of the measures Z_D are naturally interrelated; restrictions to subdomains are governed by a Gibbs-Markov property and images under analytic bijections by a conformal covariance transformation rule. Time permitting, I will also present conditions that determine the laws of these measures uniquely. All but one of these are known to hold for the critical Liouville Quantum Gravity measure associated with the Continuum Gaussian Free Field. Joint work with M. Biskup.

Rémi Rhodes (Université Paris-Est Marne La Vallée)

Liouville quantum gravity on the Riemann sphere

This talk will be concerned with the rigorous construction of 2d-Liouville Quantum Gravity (hence with positive cosmological constant) on the Riemann sphere introduced in the seminal work by Polyakov in 1981. We will also explain some of its fundamental properties like conformal covariance under $PSL_2(C)$-action, Seiberg bounds, KPZ scaling laws, KPZ formula and the Weyl anomaly (Polyakov-Ray-Singer) formula for Liouville Quantum Gravity.

Vincent Vargas (Ecole normale supérieure de Paris)

Some new estimates on the Liouville heat kernel

Liouville quantum gravity (LQG) is a conformal field theory conjectured to be the scaling limit of large random planar maps properly embedded in the sphere. For instance, the area measure of planar maps is expected to converge to the natural area measure associated to LQG, the so-called Liouville measure (formally) defined by the exponential of the Gaussian Free Field. Liouville Brownian motion (LBM) is the natural diffusion process associated to the Liouville measure. In this talk, I will consider the Liouville heat kernel, defined as the density of the LBM with respect to the Liouville measure. More precisely, I will present regularity estimates and non trivial off-diagonal bounds for the Liouville heat kernel. Based on joint work with P. Maillard, R. Rhodes and O. Zeitouni.

Ofer Zeitouni (Weizmann Institute of Science)

Extremal processes and freezing

I will discuss the relation between different notions of freezing and the structure of extremal point processes. The talk is based on ArXiv:1404.7346 (joint work with Eliran Subag)

Olivier Zindy (Université Pierre et Marie Curie, Paris)

Log-correlated Gaussian fields: study of the Gibbs measure

Gaussian fields with logarithmically decaying correlations, such as branching Brownian motion and the two-dimensional Gaussian free field, are conjectured to form universality class of extreme value statistics (notably in the work of Carpentier & Ledoussal and Fyodorov & Bouchaud). This class is the borderline case between the class of IID random variables, and models where correlations start to affect the statistics. In this talk, I will describe a general approach based on rigorous works in spin glass theory to describe features of the Gibbs measure of these Gaussian fields. I will focus on the two-dimensional discrete Gaussian free field. At low temperature, we show that the normalized covariance of two points sampled from the Gibbs measure is either 0 or 1. This is used to prove that the joint distribution of the Gibbs weights converges in a suitable sense to that of a Poisson-Dirichlet variable. (joint work with L.-P. Arguin).