Mean field games with Wright-Fisher common noise

Alekos Cecchin
(École Polytechnique)

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Abstract

Motivated by restoration of uniqueness in finite state mean field games, we introduce a common noise which is inspired by Wright-Fisher models in population genetics. Thus we analyze the master equation of this mean field game, which is a degenerate parabolic second-order partial differential equation set on the simplex whose characteristics solve the stochastic forward-backward system associated with the mean field game. We show that this equation, which is a non-linear version of the Kimura type equation studied in Epstein and Mazzeo (AMS, 2013) has a unique smooth solution whenever the normal component of the drift at the boundary is strong enough. This is enough to conclude that the mean field game with such type of common noise is uniquely solvable. Then we introduce the finite player version of the game and show that N-player Nash equilibria converge towards the solution of such a kind of Wright-Fisher mean field game. The analysis is more subtle than in the standard setting because the mean field interaction between the players now occurs through a weighted empirical measure. In other words, each player carries its own weight, which hence may differ from 1/N and which, most of all, evolves with the common noise. Finally, we give an idea on how the randomly forced and uniquely solvable mean field game is used to provide a selection principle for potential mean field games on a finite state space and, in this respect, to show that equilibria that do not minimize the corresponding mean field control problem should be ruled out. Our strategy is a tailor-made version of the vanishing viscosity method for partial differential equations. Here, the viscosity has to be understood as the intensity of a the Wright-Fisher common noise. Based on joint works with Erhan Byraktar, Asaf Cohen, and François Delarue.