The stationary marked random connection model: uniqueness of the infinite cluster and sharp phase transition

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— Berlin Probability Colloquium —

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Abstract

We consider a Poisson process η on $\mathbb{R}^d \times \mathbb{X}$, whose intensity measure is a multiple of the product of Lebesgue measure and a distribution on the mark space $\mathbb{X}$. Pairs of (marked) Poisson points are connected with a probability that depends on the points in a symmetric and translation invariant way.

The random connection model (RCM) is the resulting random graph with vertex set η. It is stationary and ergodic with respect to space shifts. Well-known special cases are the Boolean model with general compact grains and the so-called weighted RCM.

Percolation occurs if there is at least one infinite cluster (component). We prove that the infinite clusters are deletion stable, that is the removal of a Poisson point cannot split a cluster in two or more infinite clusters. We prove that this stability together with a natural irreducibility assumption implies uniqueness of the infinite cluster. This extends and unifies several results in the literature.

In the second part of the talk we will discuss sharp phase transition. Under additional assumptions on the connection function, the size of the typical cluster in the subcritical regime is exponentially small.

The talk is based on recent joint work with Mikhail Chebunin (Karĺsruhe).

*Punctual, i.e. sine tempore!