Exploring the Weather-Climate Connection: Probabilistic Climate Dynamics

Part II: Stochastic Climate Models

Adam Monahan

monahana@uvic.ca

School of Earth and Ocean Sciences, University of Victoria

Overview

- Why is climate complex?

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- Modelling the climate system

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- Origins of stochasticity in the climate system.

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- Case Study I: Stochastic dynamics of sea-surface temperature.

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- Case Study II: Stochastic dynamics of sea-surface winds.

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Introduction: Stochastic Climate Models

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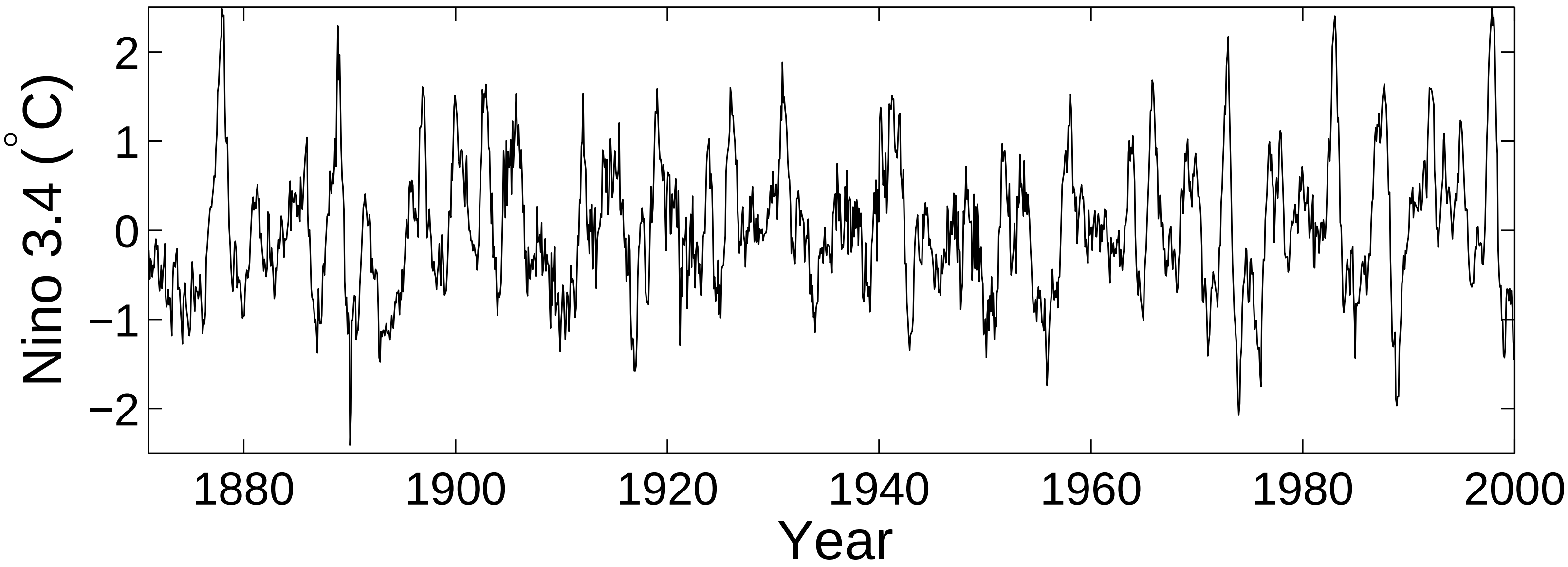

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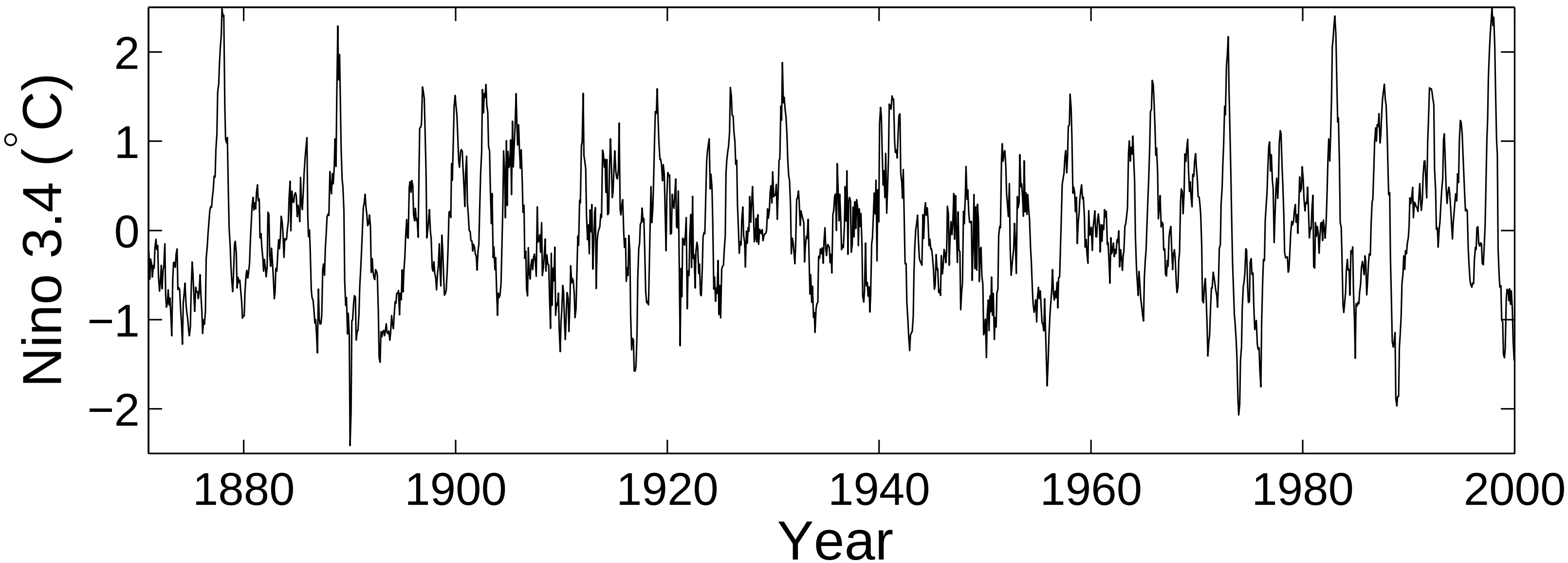

- Natural to model dynamics as interaction between **deterministic** and **random** (turbulent) parts of flow $\Rightarrow$ stochastic climate models

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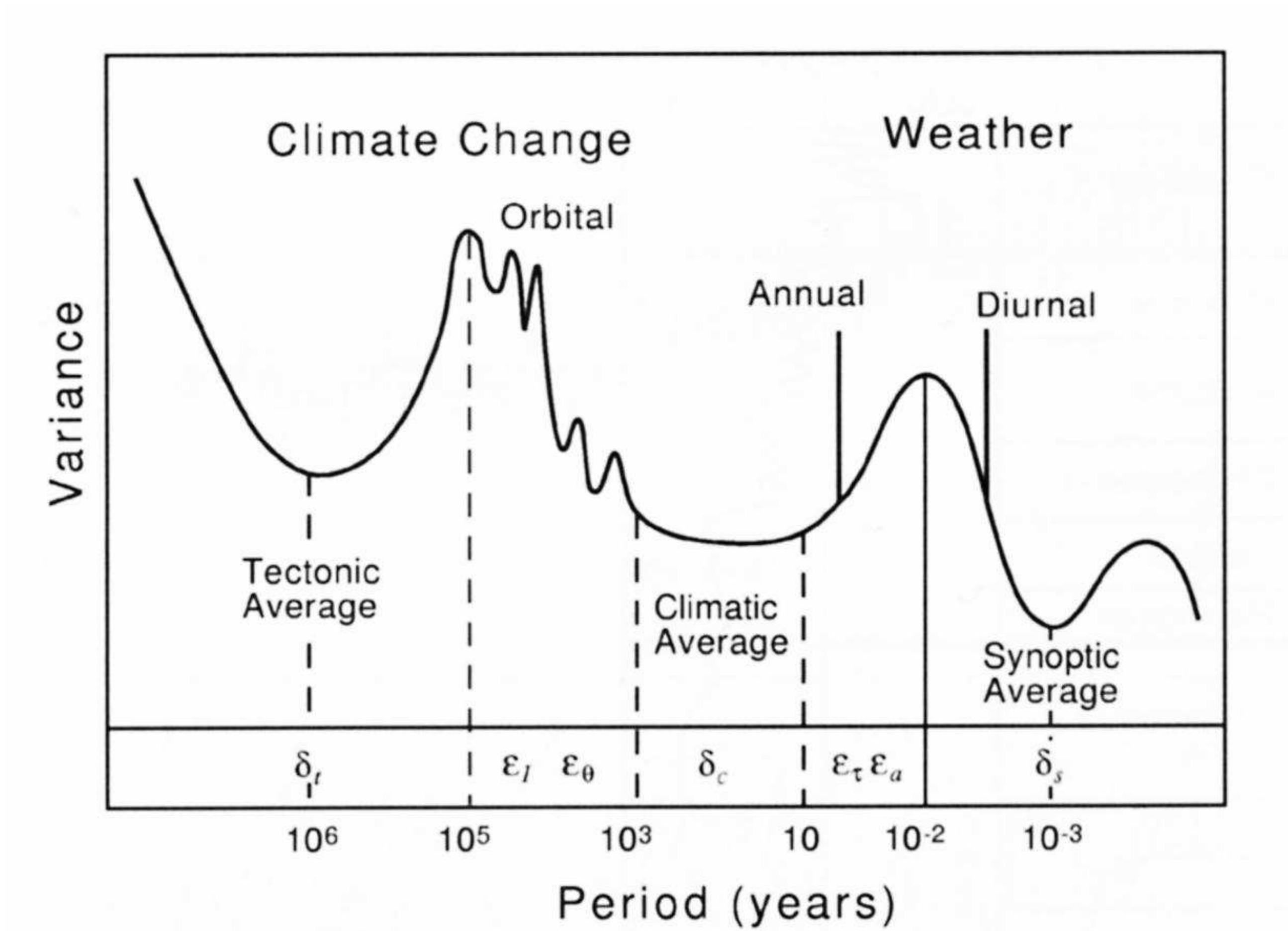
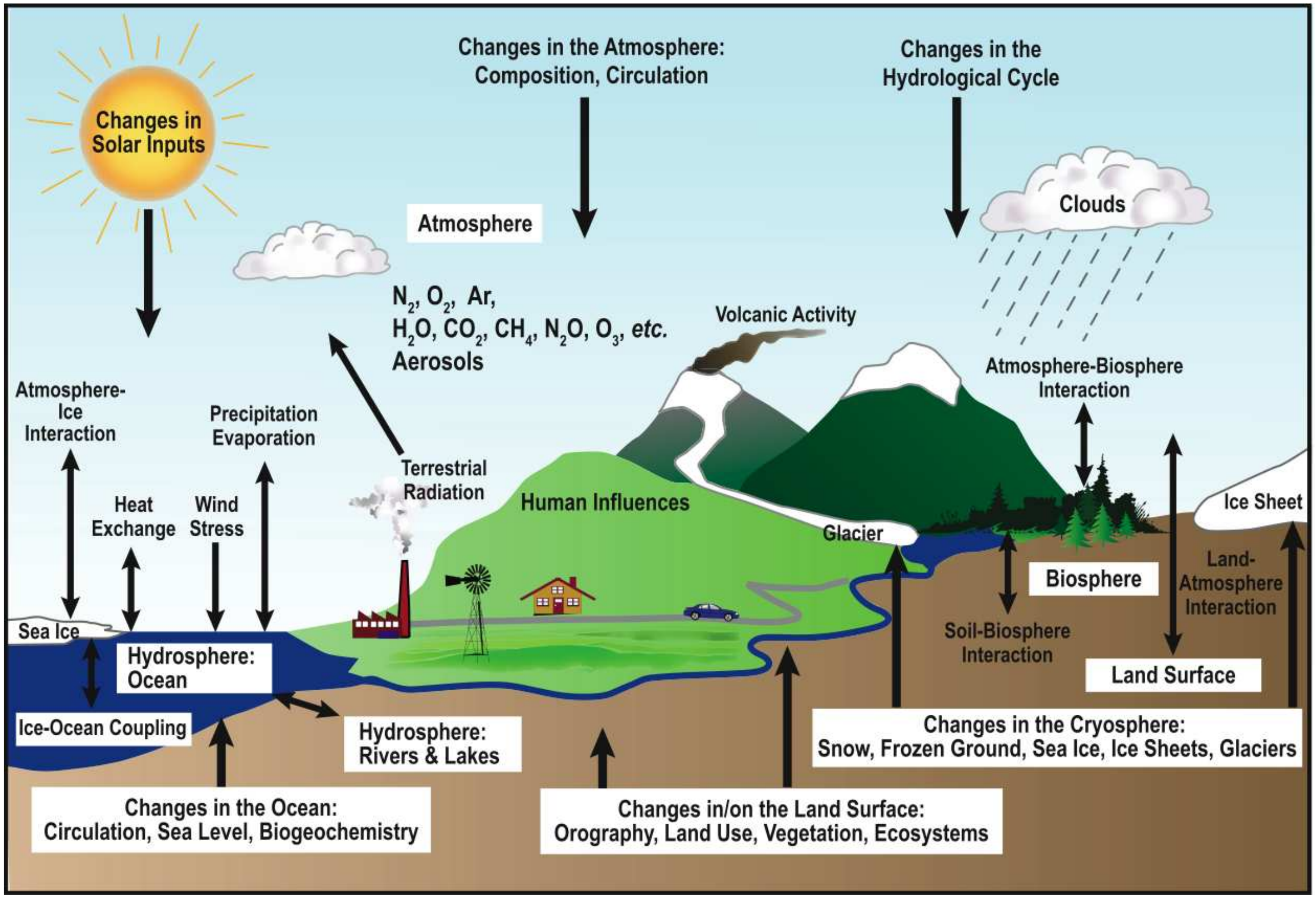
Why is Climate Complex? Coupling Across Scales

- Climate system displays variability over broad range of space and time scales



From Saltzman, 2002

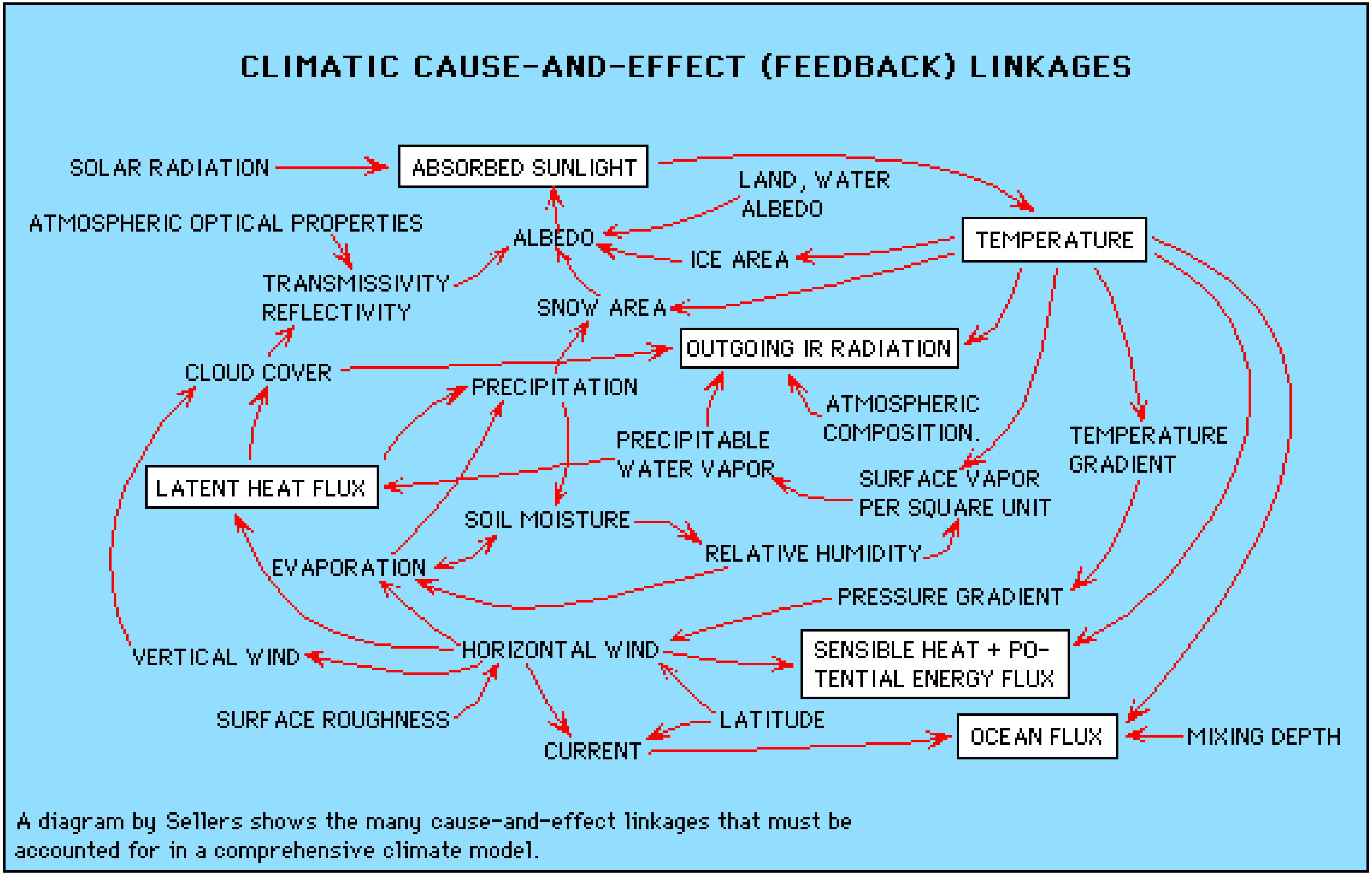
Why is Climate Complex? Coupling Across Systems



From IPCC AR4 http://www.ipcc.ch

FAQ 1.2, Figure 1

Why is Climate Complex? Feedback Loops



From `http://eesc.columbia.edu/courses/ees/slides/climate/`

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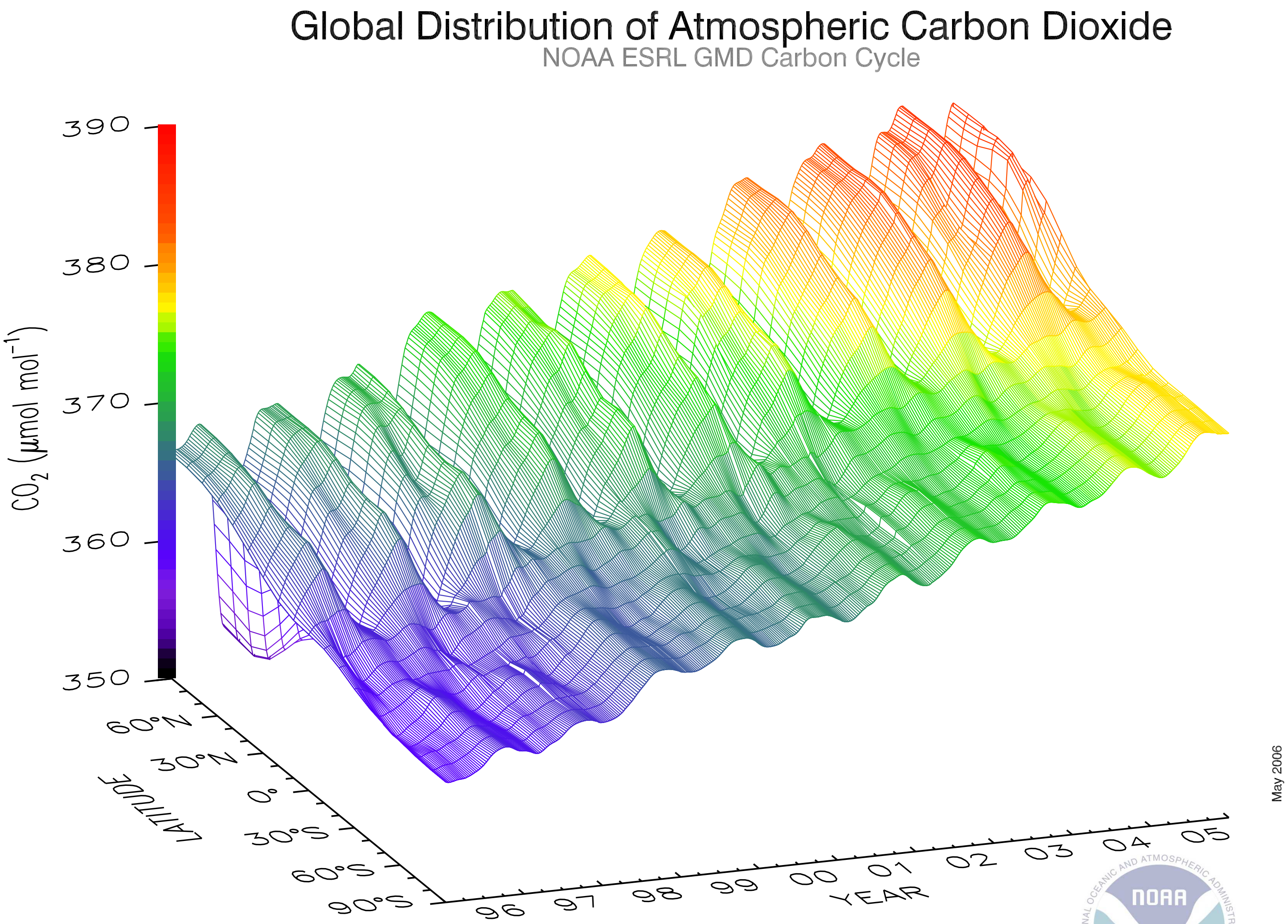
Why is Climate Complex? Non-Stationarity



Three dimensional representation of the latitudinal distribution of atmospheric carbon dioxide in the marine boundary layer. Data from the GMD cooperative air sampling network were used. The surface represents data smoothed in time and latitude. Contact: Dr. Pieter Tans and Thomas Conway, NOAA ESRL GMD Carbon Cycle, Boulder, Colorado, (303) 497-6678 (pieter.tans@noaa.gov, http://www.cmdl.noaa.gov/ccgg).

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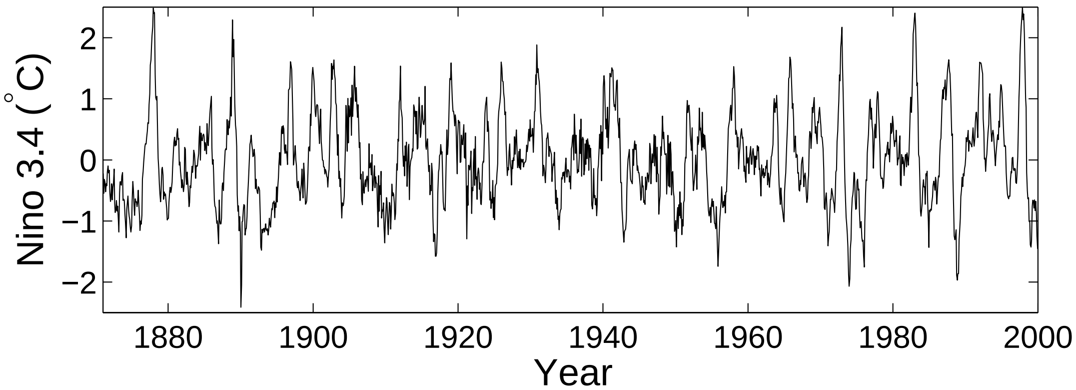
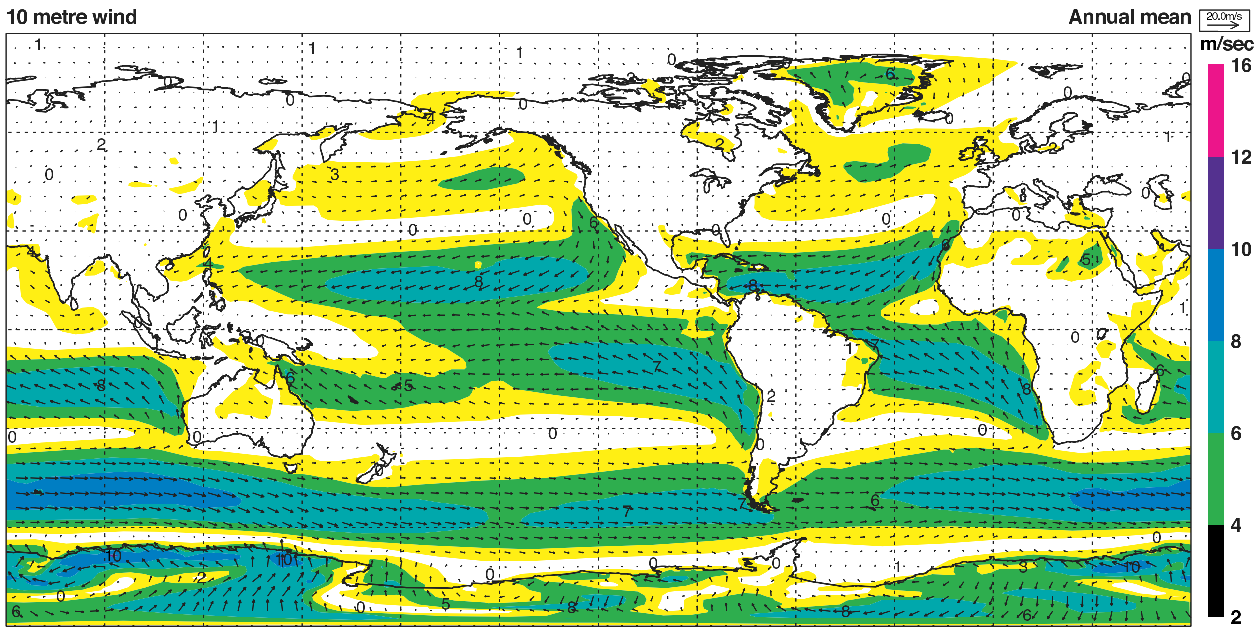
Why is Climate Complex? Data: Too Much & Not Enough





From ERA-40 Project Report Series 19

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Modelling the Climate System

- Climate models represent physical (+ biogeochemical) processes, typically through consideration of **budgets** of **“conserved quantities”**

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$\Rightarrow$ strong physical coupling of processes across different space and time scales

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$\Rightarrow$ "subgrid-scale parameterisations" (closure)

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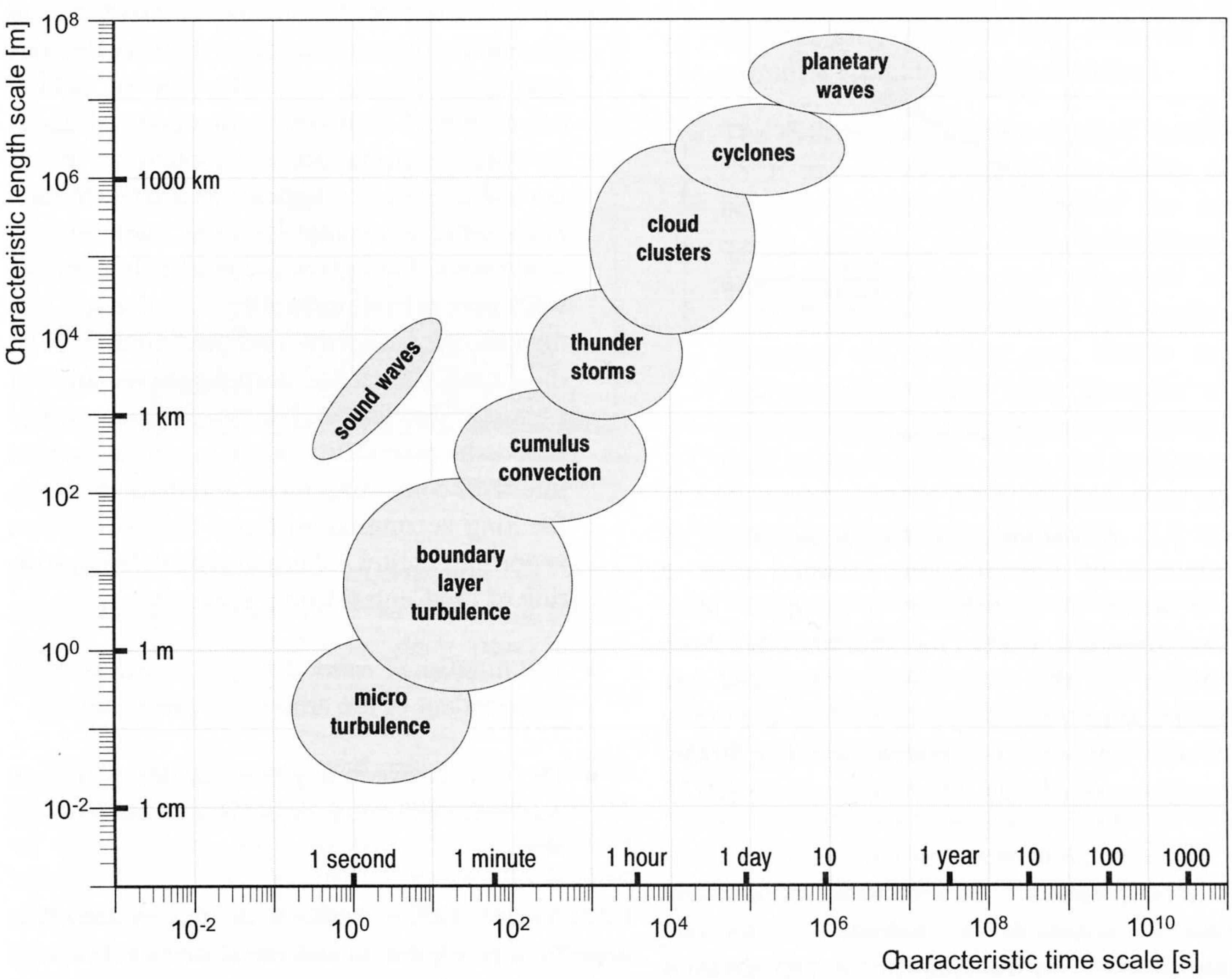

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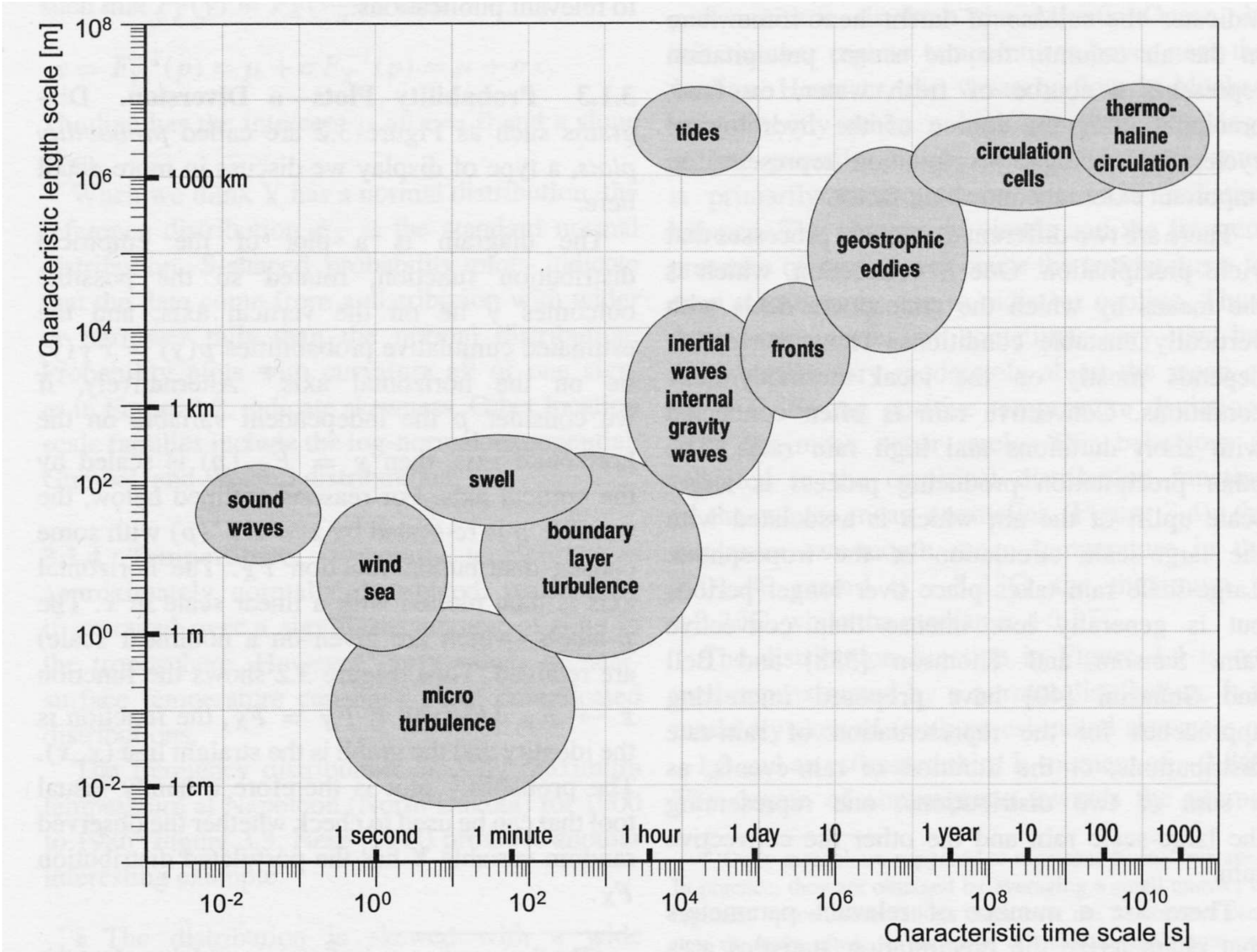

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- “coarse graining" may involve scale separations in space, time, or both
- coarse-graining results not only in unresolved *scales*, but also unresolved *processes* (e.g. internal gravity waves, convection, cloud mircophysics)

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$$
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$$\frac{d}{dt}\zeta = [D\overline{f}(\overline{\mathbf{x}})]\zeta + \sigma(\overline{\mathbf{x}})\dot{\mathbf{W}}$$

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3. **“Hasselmann approximation”** (stochastic)
$$\frac{d}{dt}\overline{\mathbf{x}} = \overline{f}(\overline{\mathbf{x}}) + \sqrt{\epsilon}\sigma(\overline{\mathbf{x}}) \circ \dot{\mathbf{W}}$$

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Origins of Stochasticity: Multiscale Dynamics

- Special case of averaging considered by Majda, Timofeyev, and Vanden-Eijnden (MTV Theory)

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- In many (most?) climate applications, ϵ is not small
- Fast/slow decomposition not unique;
  “one person’s noise is another person’s signal”

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  - Building large ensembles computationally expensive

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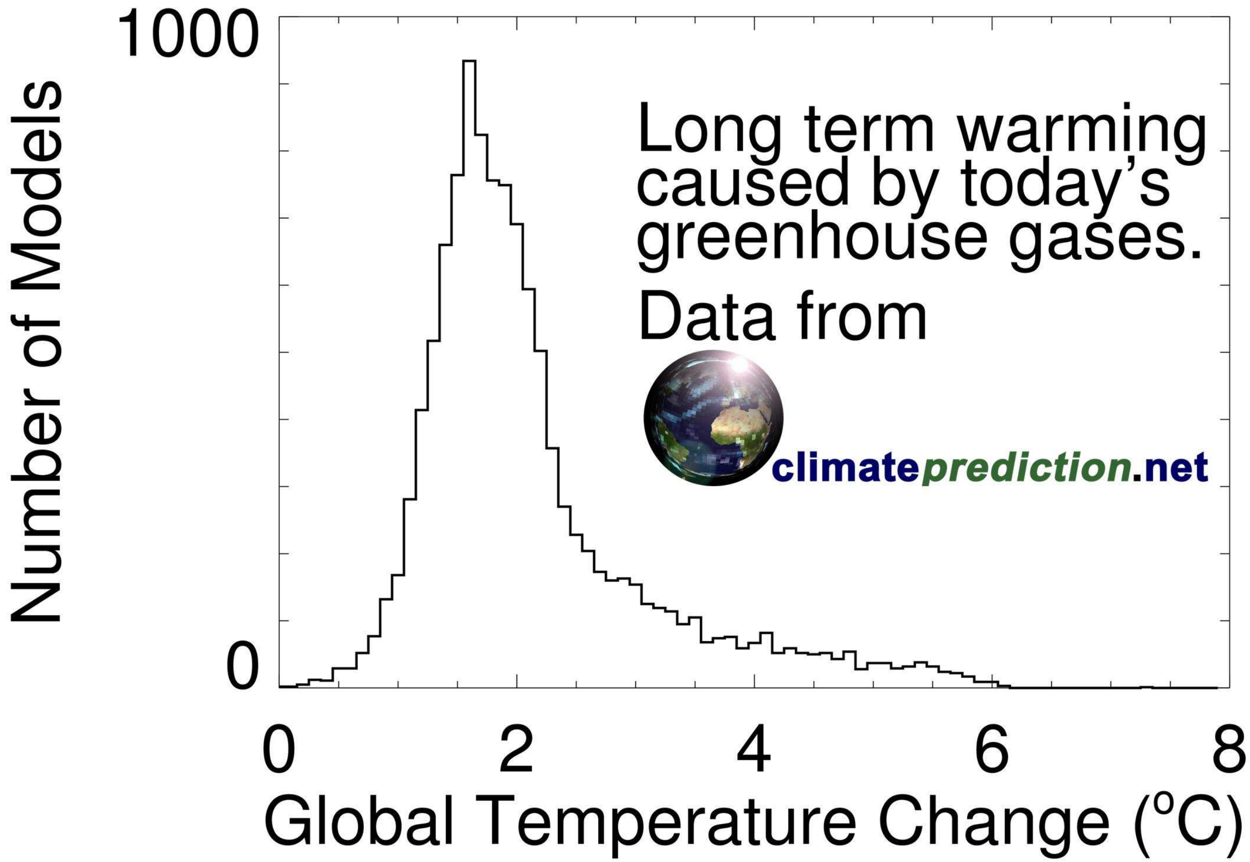
Parameter Uncertainty: Ensemble Prediction

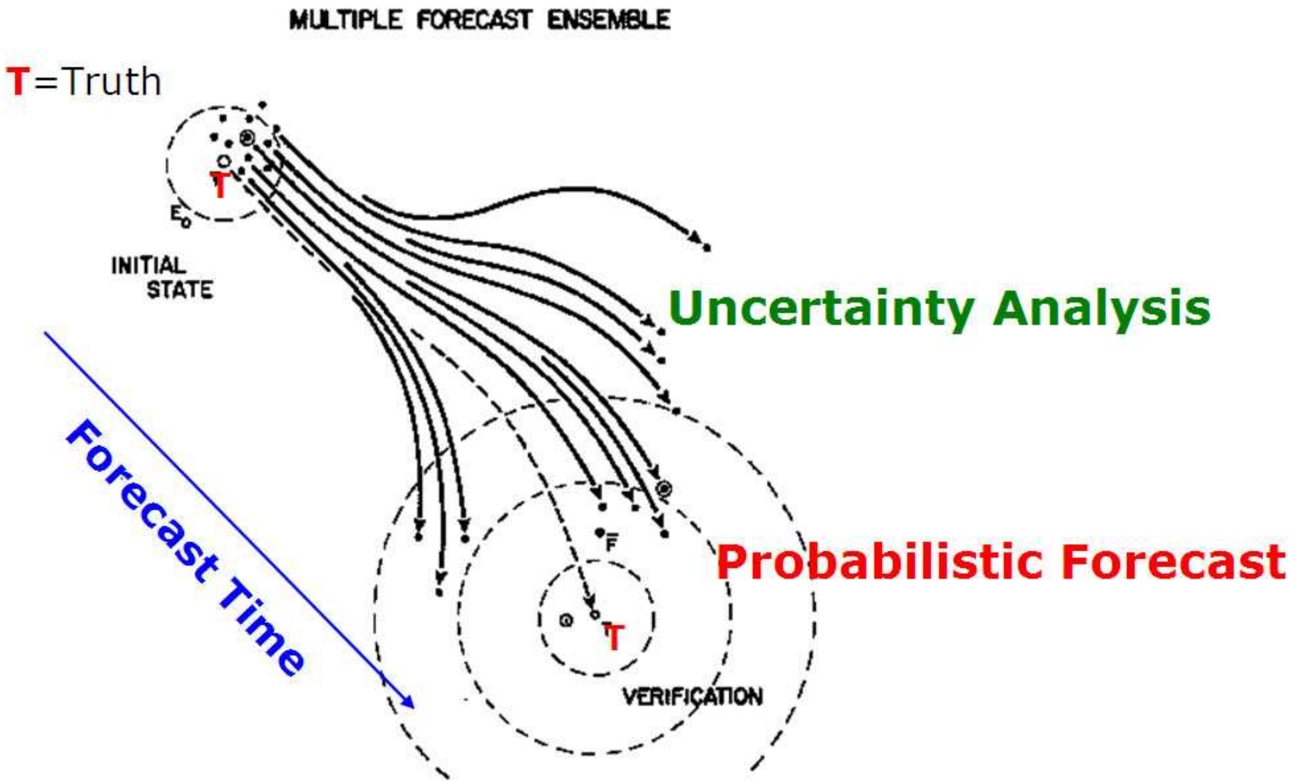
- `climateprediction.net` uses idle private CPUs to integrate ensembles with different parameter settings



`http://www.climateprediction.net`

Initial Condition Uncertainty: Ensemble Forecasting

- Model uncertainties can also include initial conditions



http://chrs.web.uci.edu/images/ensemble_large_atmo.jpg

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  - rates of biological activity sensitive to ocean temperature

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Bulk Surface Mixed Layer Model

- Dynamics of local SST:

$$\frac{\partial T}{\partial t} = -\mathbf{v}\cdot\nabla T + \frac{F_{sh}(t)}{h(t)} + \frac{F_{lh}(t)}{h(t)} + \frac{\overline{w'T'}\big|_{z=h}(t)}{h(t)} + \frac{F_{sw}(t)}{h(t)} - \sigma(\epsilon T^4 - \epsilon_a T_a^4)$$

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where

T	sea-surface temperature (SST)
$-\mathbf{v}\cdot\nabla T$	horizontal advective tendency
$h(t)$	mixed layer (ML) depth
$F_{sh}(t) = c_h\|\|\mathbf{u}\|\|(T_a - T)$	surface "sensible heat" flux
$F_{lh}(t) = c_h\|\|\mathbf{u}\|\|(q_a - q_s(T))$	surface "latent heat" flux
$\overline{w'T'}\big|_{z=h}(t)$	turbulent fluxes at ML base
$F_{sw}(t)$	surface shortwave (solar) heating
$\sigma(\epsilon T^4 - \epsilon_a T_a^4)$	net surface longwave cooling

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Local stochastic model

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$\Rightarrow$ simple Ornstein-Uhlenbeck process

$$\frac{dT'}{dt} = -\frac{1}{\tau}T' + \gamma\dot{W}$$

with spectrum

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- “Slow” local surface ocean dynamics $\Rightarrow$ red-noise response to “fast” atmospheric forcing

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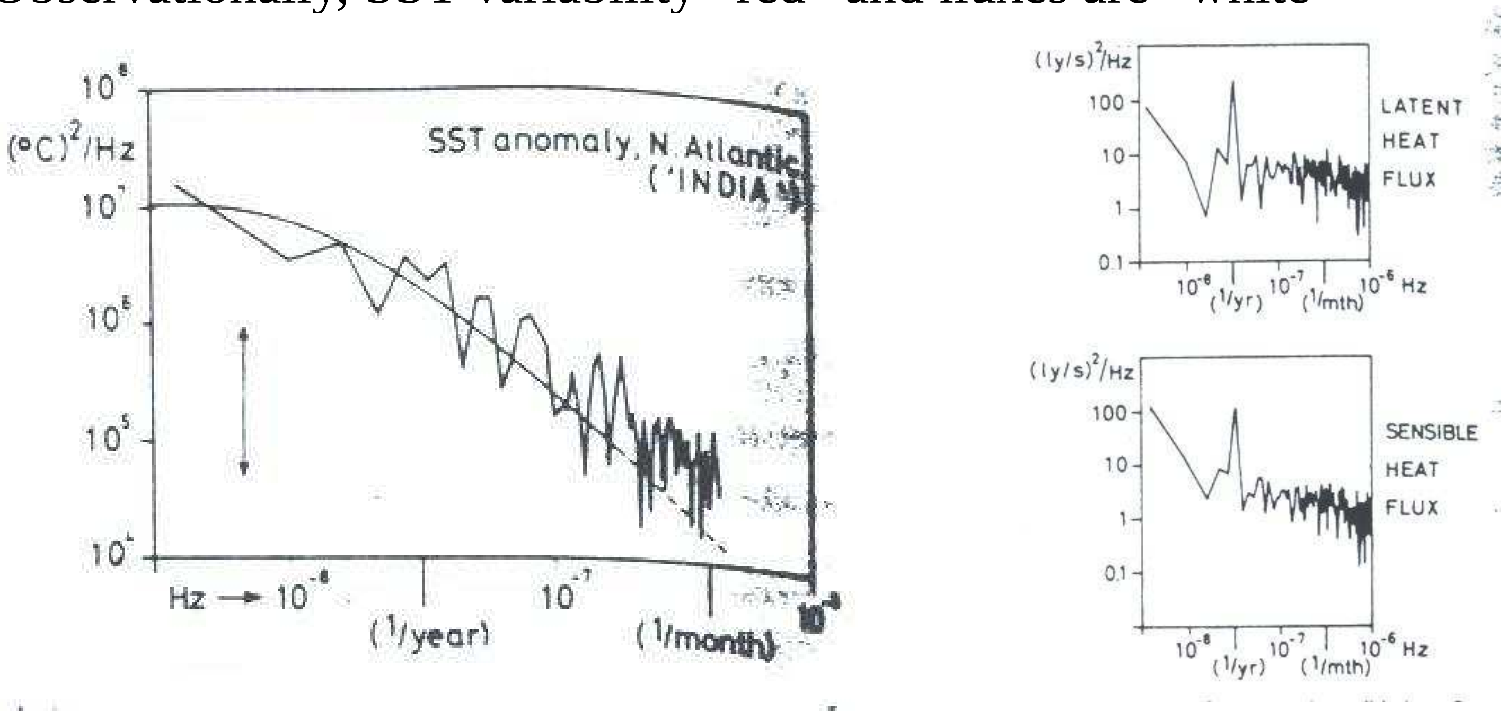
What do we learn about SST variability?

- Observationally, SST variability “red” and fluxes are “white”



From Frankignoul & Hasselmann *Tellus* 1977

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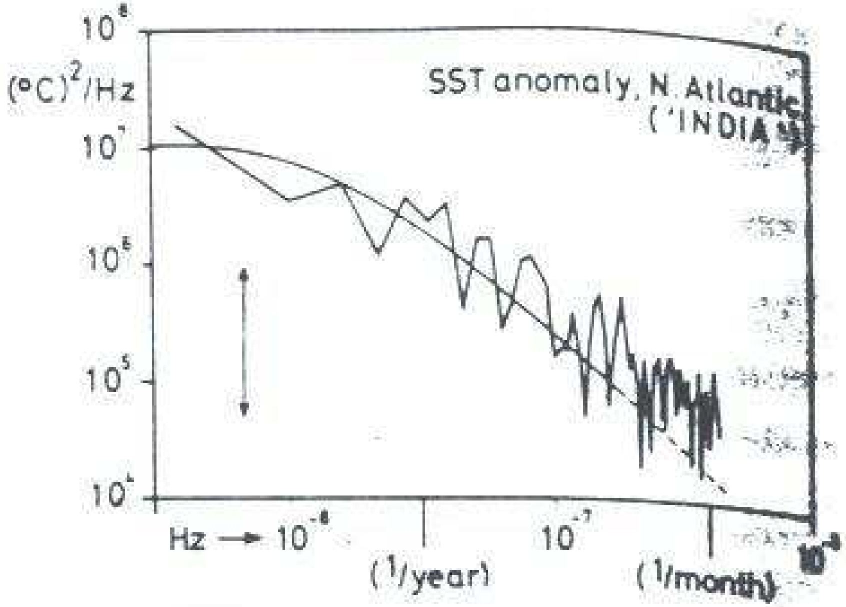

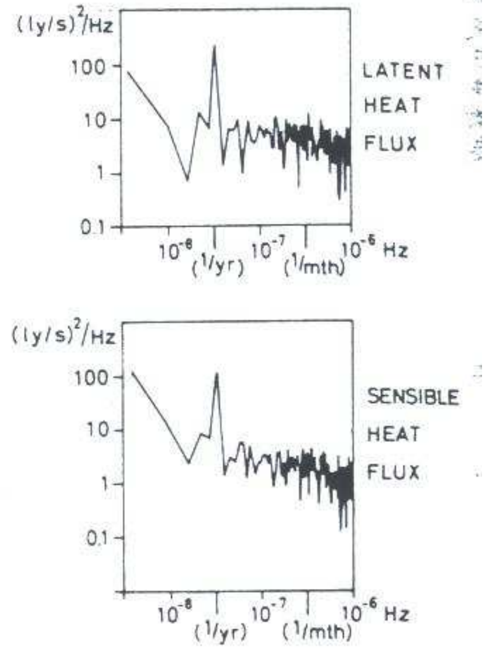

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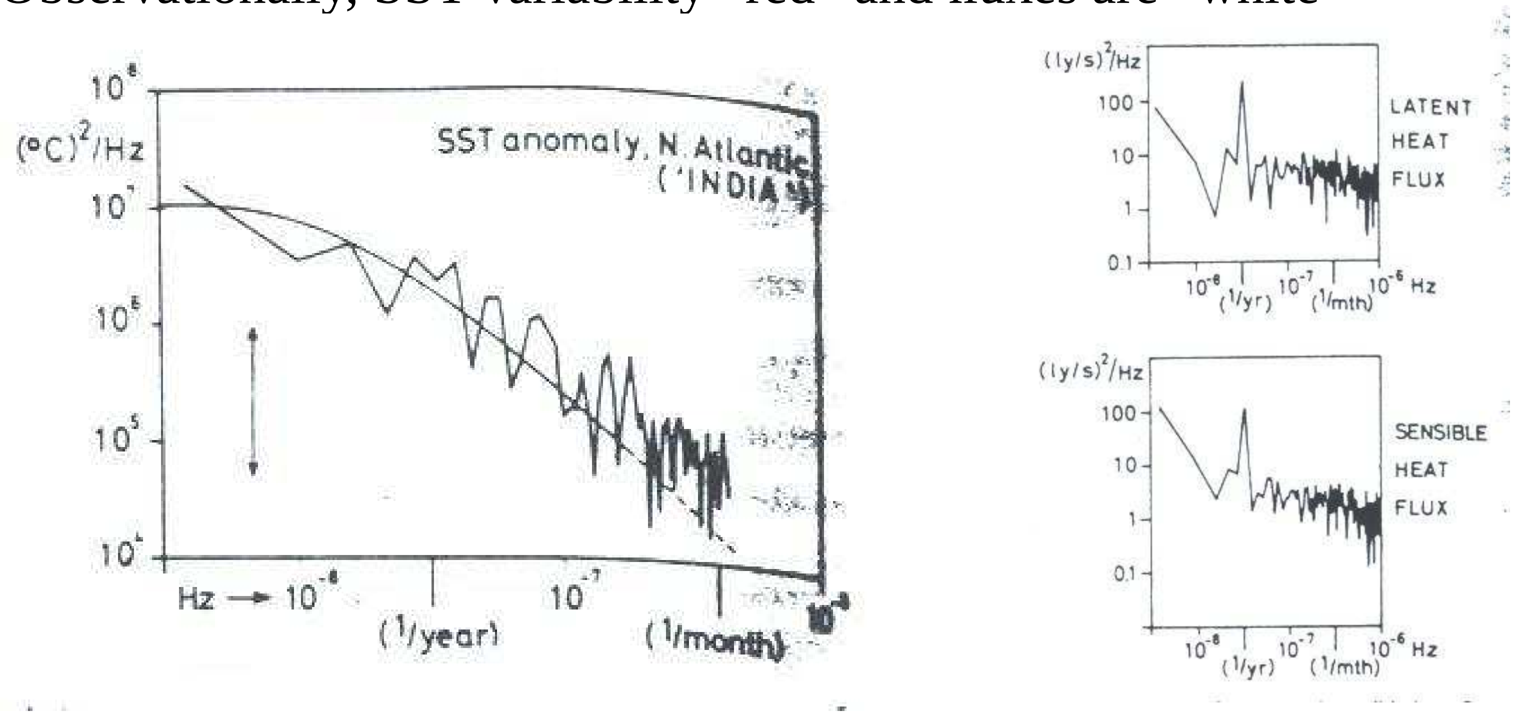

From Frankignoul & Hasselmann *Tellus* 1977

- Linear stochastic model ⇒ simple null hypothesis for observed variability
- Generalisation with multiplicative noise effects explains slight non-Gaussianity of SST

(c.f. Sura, Newman, & Alexander *J. Phys. Oceanogr.*, 2006)

Sea Surface Winds: Why Should We Care?

- *Air/Sea Exchange*

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  - generation rate scales as cube of wind speed; extreme events important

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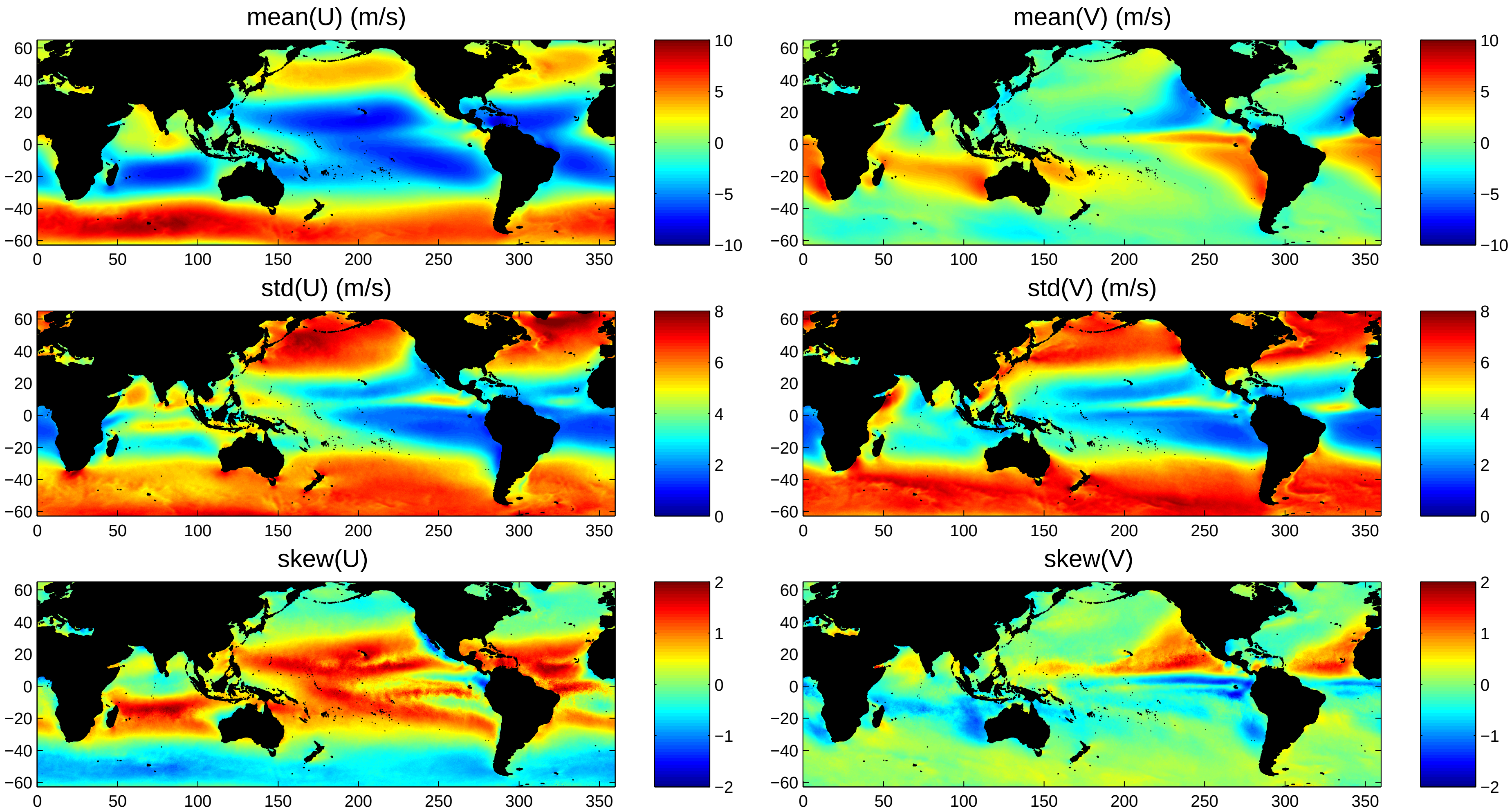
Vector Wind Moments



Zonal Wind

Meridional Wind

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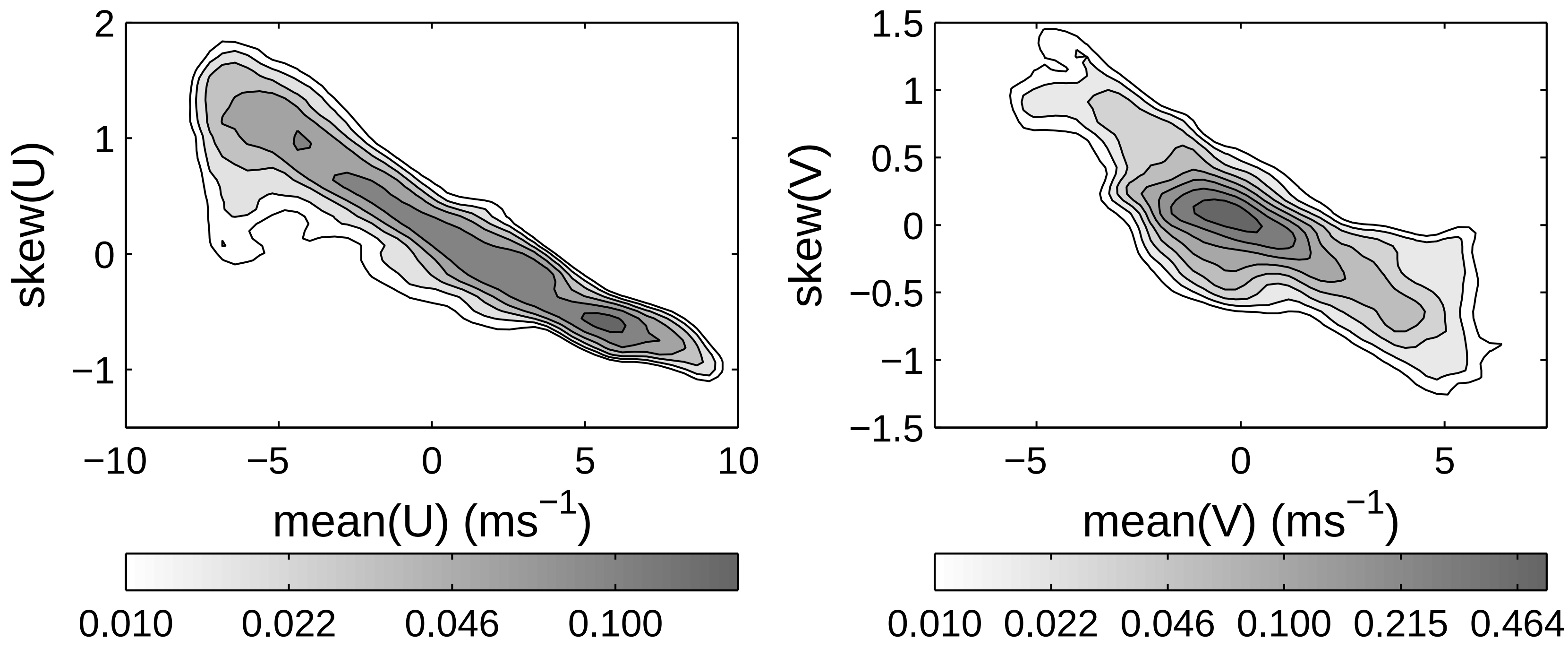
Mean and Skewness of Vector Wind

- Joint pdfs of mean and skew for zonal and meridional winds



(note logarithmic contour scale)

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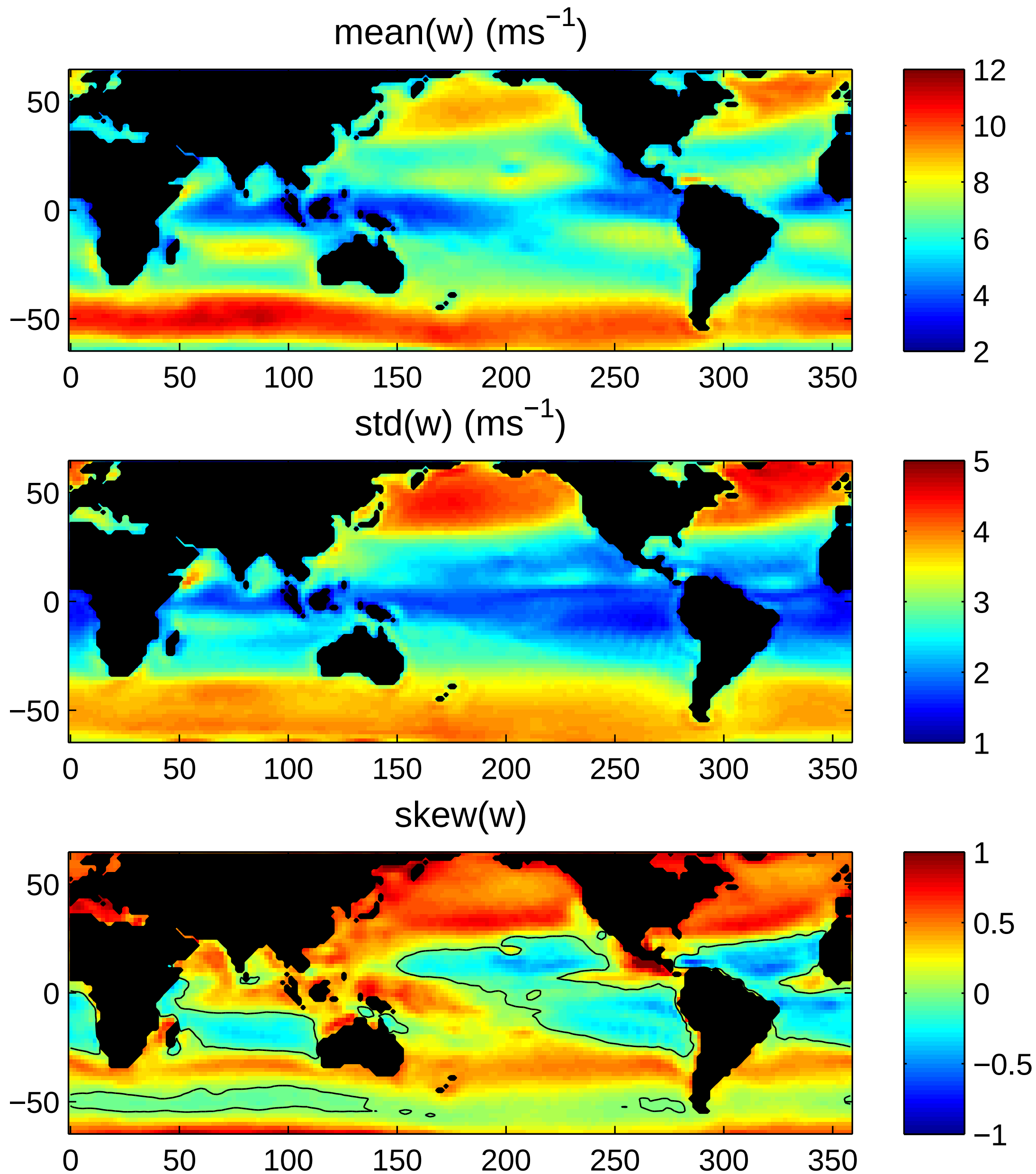
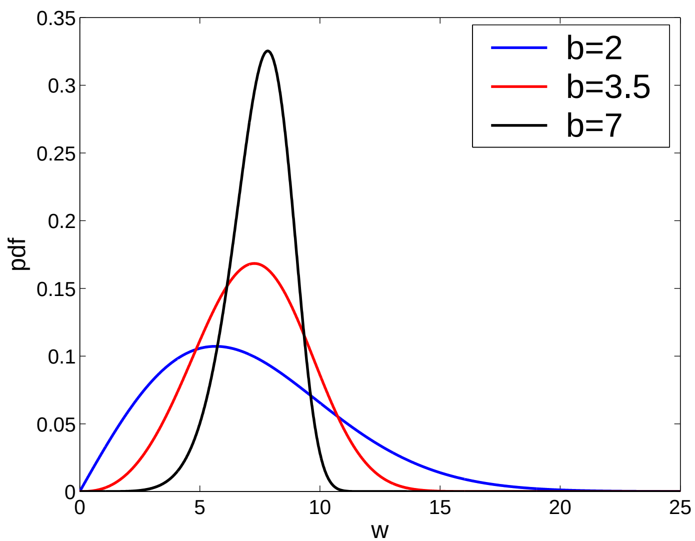
Wind Speed Moments



Wind Speed pdf: Weibull distribution

- The pdf of wind speed w has traditionally (and empirically) been represented by 2-parameter Weibull distribution:



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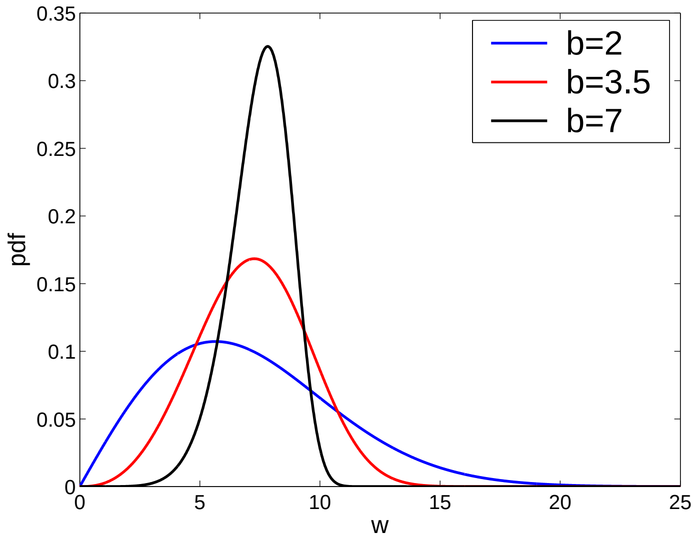

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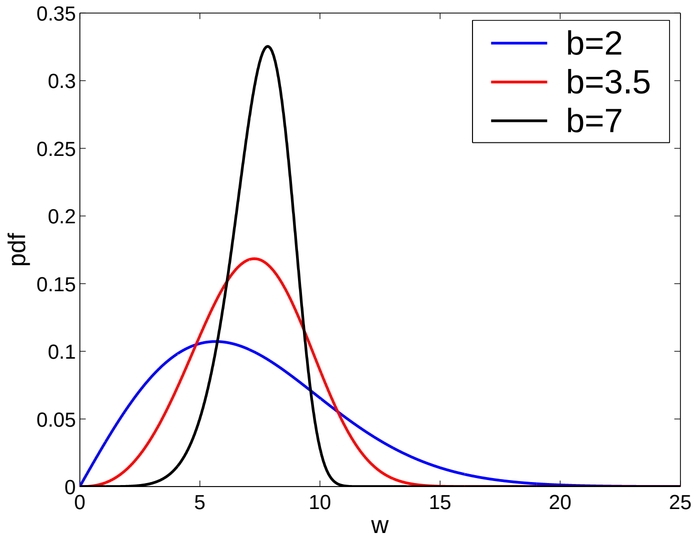

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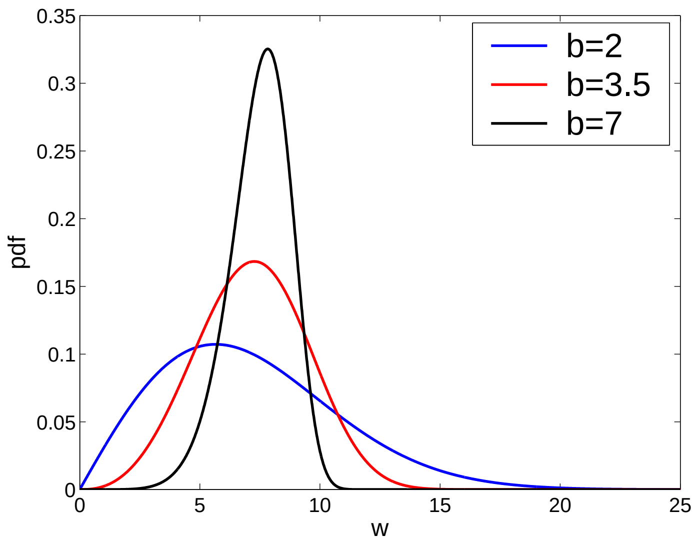

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- $p_w(w)$ is unimodal

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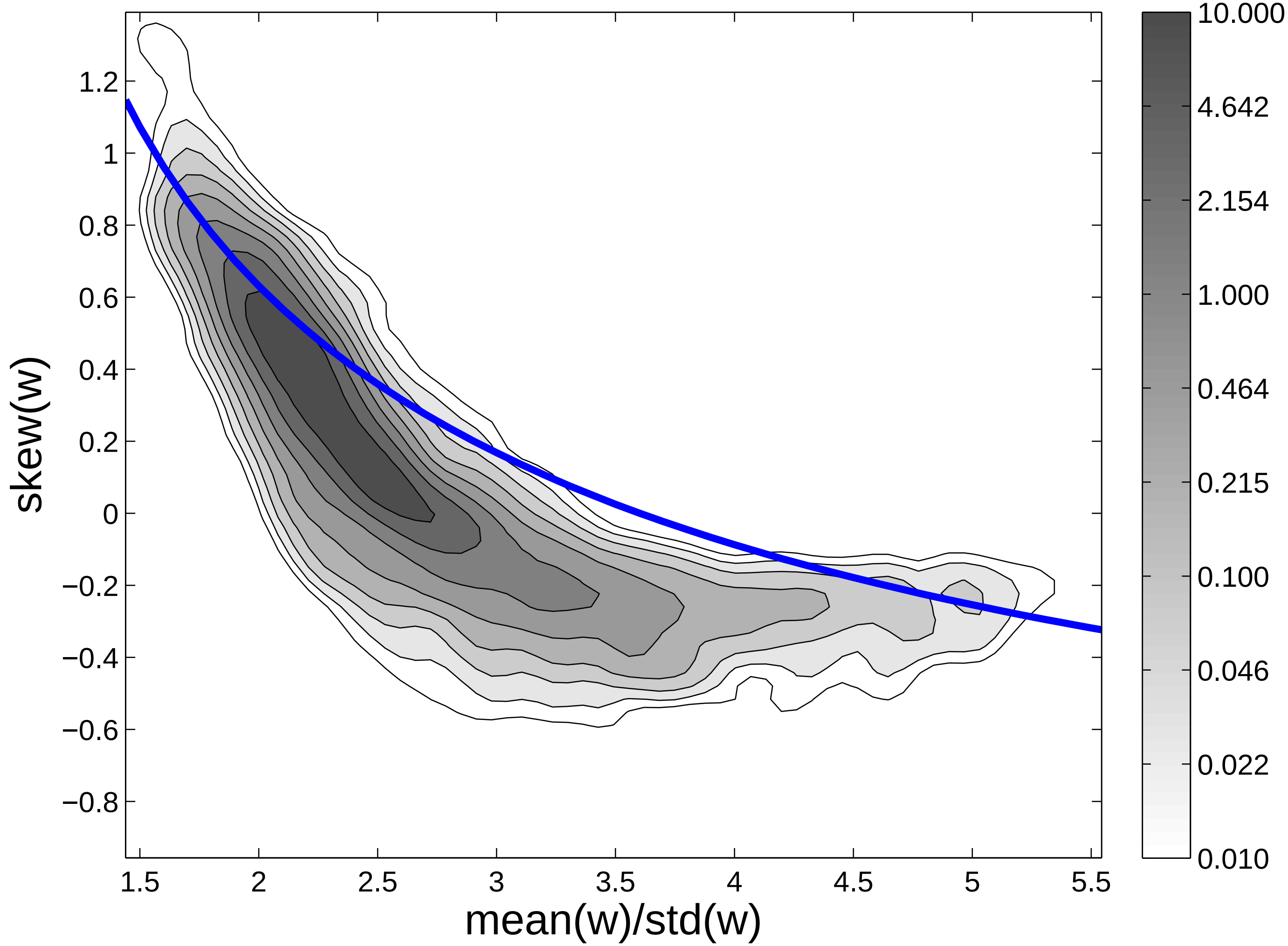
Wind Speed pdfs: Observed

- Observed speed moments fall around Weibull curve



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Boundary Layer Dynamics

- Horizontal momentum equations:

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$$\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} = -\frac{1}{\rho} \nabla p - f \hat{\mathbf{k}} \times \mathbf{u} - \frac{1}{\rho} \frac{\partial (\rho \overline{\mathbf{u}' u_3'})}{\partial z}$$

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$$\frac{d\mathbf{u}}{dt} = -\frac{1}{\rho}\nabla p - f\hat{\mathbf{k}} \times \mathbf{u} + \frac{1}{h}\left(\overline{\mathbf{u}'u_3'}(0) - \overline{\mathbf{u}'u_3'}(h)\right)$$

Surface Wind Stress

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where

$$\begin{aligned} u &= \text{along-mean wind component} \\ v &= \text{cross-mean wind component} \\ \mathbf{u} &= (u, v) \\ u_3 &= \text{vertical wind component} \end{aligned}$$

Surface Wind Stress

- Surface wind stress is turbulent momentum flux across air/sea interface:

$$\tau_s = \rho_a \overline{\mathbf{u}' u_3'}(0)$$

where
$$\begin{aligned} u &= \text{along-mean wind component} \\ v &= \text{cross-mean wind component} \\ \mathbf{u} &= (u, v) \\ u_3 &= \text{vertical wind component} \end{aligned}$$

- Flux parametrised in terms of $\mathbf{u}$ by bulk drag formula:

$$\tau_s = \rho_a c_d w \mathbf{u}$$

where $w = \parallel \mathbf{u} \parallel$ is the wind speed.

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Surface Momentum Budget

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$$\begin{aligned}
\frac{d\mathbf{u}}{dt} &= -\frac{1}{\rho}\nabla p - f\hat{\mathbf{k}} \times \mathbf{u} - \frac{c_d}{h} w\mathbf{u} + \frac{W_e}{h}(\mathbf{U} - \mathbf{u}) \\
&= \mathbf{\Pi} - \frac{c_d}{h} w\mathbf{u} - \frac{W_e}{h}\mathbf{u}
\end{aligned}$$

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Mechanistic Model: SDE

- Decomposing $\mathbf{\Pi}$ into mean and fluctuations:

$$\begin{aligned}\Pi_u(t) &= \langle \Pi_u \rangle + \sigma \dot{W}_1(t) \\ \Pi_v(t) &= \sigma \dot{W}_2(t)\end{aligned}$$

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we obtain stochastic differential equation

$$\begin{aligned}\frac{du}{dt} &= \langle \Pi_u \rangle - \frac{c_d}{h} wu - \frac{W_e}{h} u + \sigma \dot{W}_1 \\ \frac{dv}{dt} &= -\frac{c_d}{h} wv - \frac{W_e}{h} v + \sigma \dot{W}_2\end{aligned}$$

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Mechanistic Model: pdf

- Solution of associated Fokker-Planck equation for stationary pdf:

$$p_{uv}(u,v) = \mathcal{N}_1 \exp\left(\frac{2}{\sigma^2}\left\{\langle \Pi_u \rangle\, u - \frac{W_e}{2h}(u^2+v^2) - \frac{1}{h}\int_0^{\sqrt{u^2+v^2}} c_d(w')w'^2\, dw'\right\}\right)$$

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- Changing to polar coordinates and integrating over angle gives wind speed pdf:

$$p_w(w) = \mathcal{N} w I_0\left(\frac{2\,\langle\Pi_u\rangle\, w}{\sigma^2}\right)\exp\left(-\frac{2}{\sigma^2}\left\{\frac{W_e}{2h}w^2 + \frac{1}{h}\int_0^w c_d(w')w'^2\, dw'\right\}\right)$$

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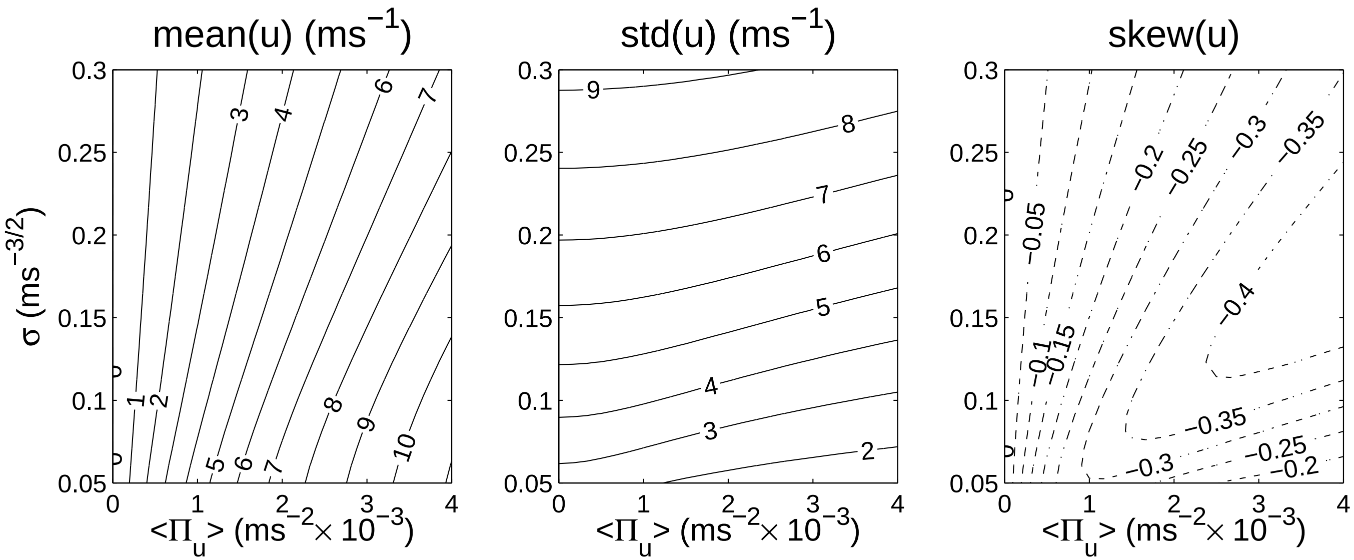
Mechanistic Model: Predictions



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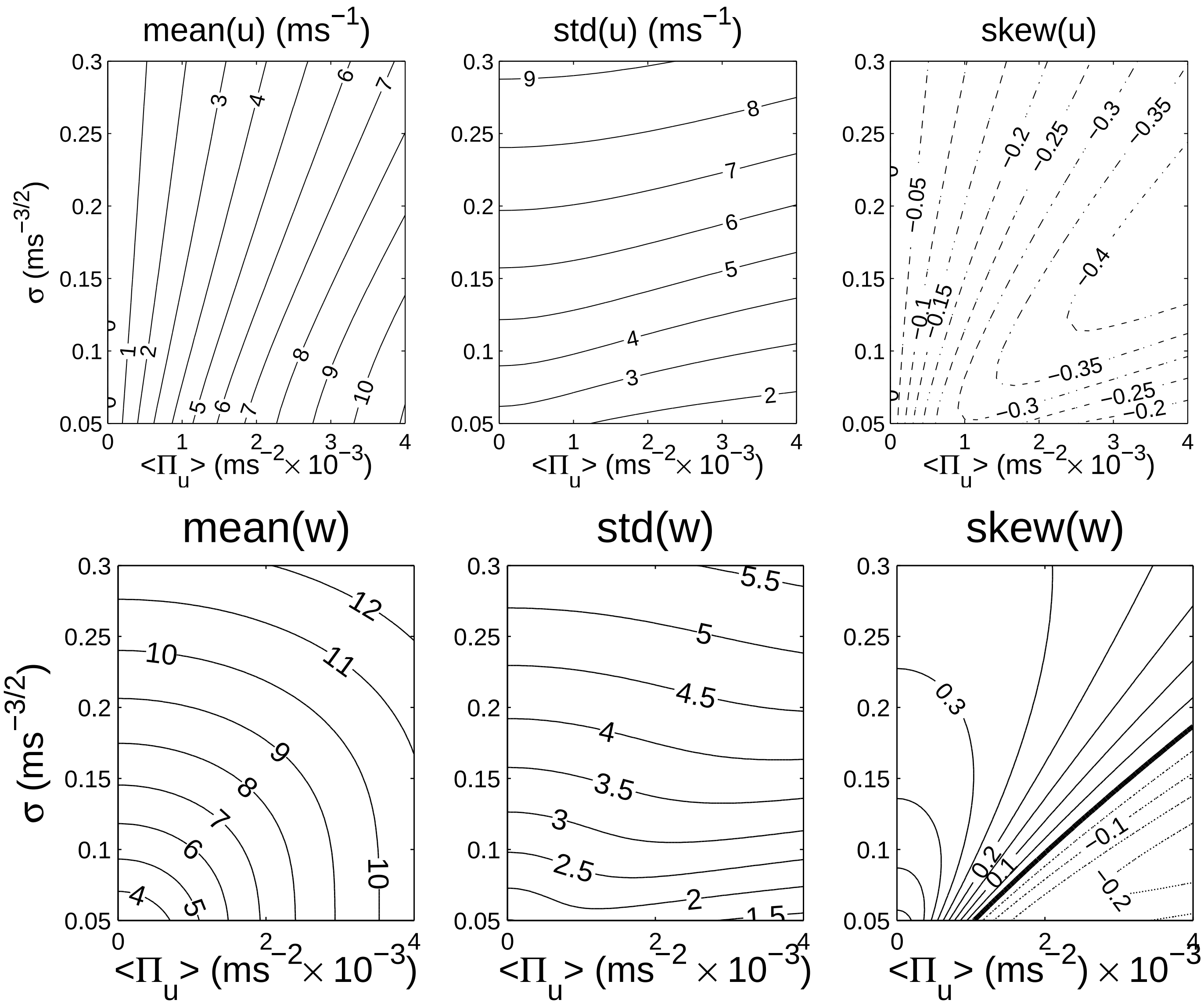
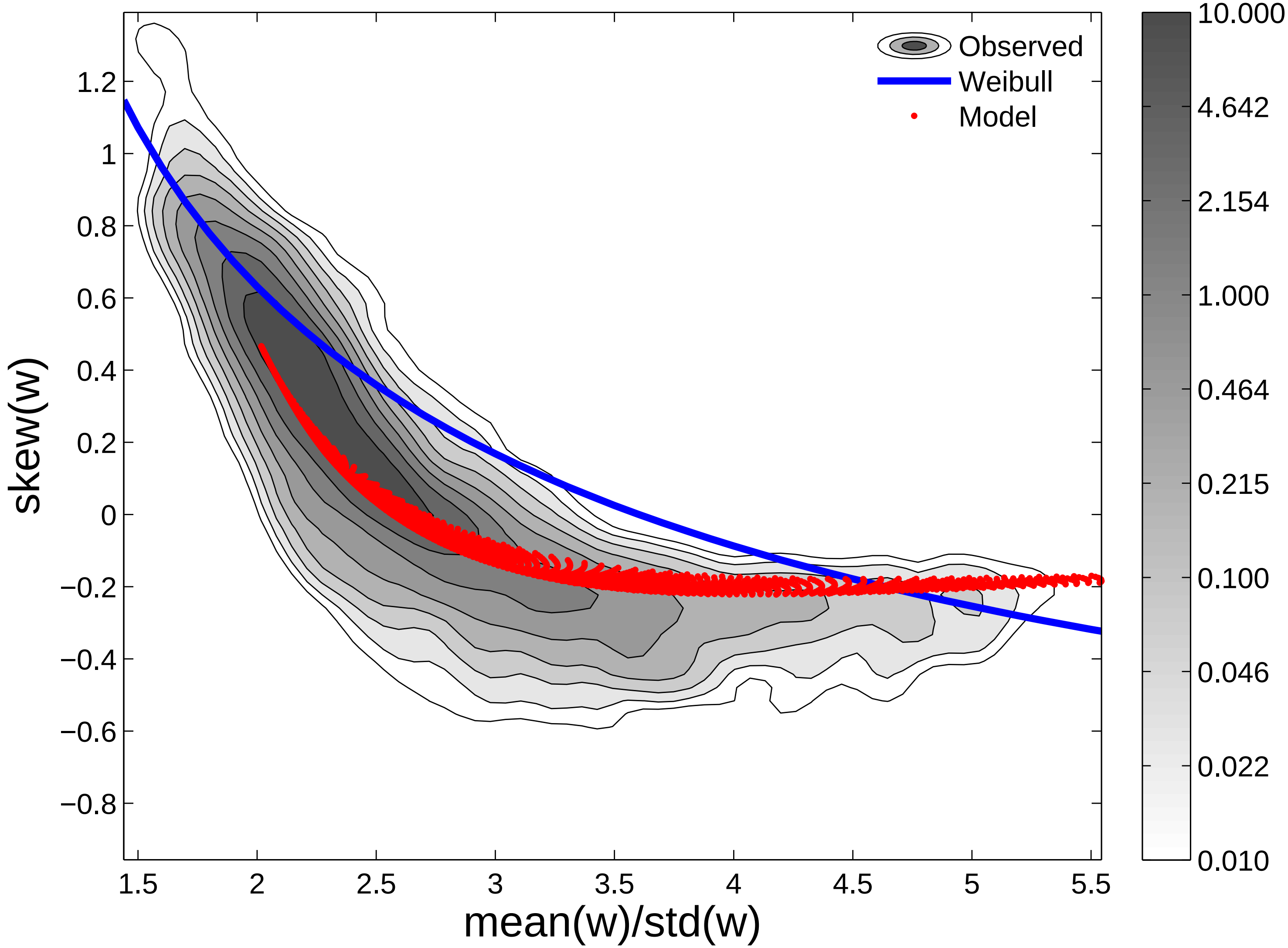


Mechanistic Model: Comparison with Observations



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What Did We Learn About Sea Surface Winds?

- Sea surface wind pdfs characterised by relationships between moments
- These moment relationships reflect physical processes producing distributions
- Idealised stochastic models can be constructed from basic physical principles to (qualitatively) explain physical origin of pdf structure
- More accurate quantitative simulation requires a more sophisticated model; qualitative utility of relatively simple model suggests it captures essential physics

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