

# Abstracts - spring meeting Bielefeld/Beijing and Berlin/Zurich

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**Christoph Berns (U Bielefeld).** *Kawasaki Dynamics of Interacting Particles in Continuum: Micro- and Mesoscopic Description*

Abstract: In this paper we investigate a (conservative) jump dynamics of interacting particles in continuum which have a (grand canonical) Gibbs measure as an invariant measure. This dynamics is an analog of the Kawasaki dynamics of lattice spin systems. We construct a non-equilibrium evolution by solving the hierarchical equations for correlation functions. Afterwards we perform a Vlasov-type scaling of the dynamics which leads to a rescaled and limiting evolution of correlation functions. The limiting evolution preserves chaos. This leads to the derivation of a kinetic equation for the particle density of the limiting system which we call Vlasov-type equation.

**Lukasz Derdziuk (U Bielefeld).** *On some classes of birth-and-death processes in continuum*

We consider two models of birth and death processes in continuum. First of them, the contact model in random environment is an example of a single-component particle system. The existence and regularity of the evolution is proved together with the corresponding evolution of correlation functions. Also, the Glauber-Potts type model is studied as an example of two-component IPS. The corresponding evolution is constructed. Moreover, we apply the Vlasov-type scaling of the dynamics and obtain the convergence results. Also, the Vlasov-type equation is derived.

**Joscha Diehl (TU Berlin).** *Robust Filtering: Correlated Noise and Multidimensional Observation*

The filtering problem is concerned with the distribution of an Ito diffusion (the signal) conditioned on another process (the observation). In the case of independent signal and observation or the case of one dimensional observation, it was shown in Clark-Crisan [On a robust version of the integral representation formula of nonlinear filtering. Probability Theory and Related Fields, 133, 2005.] that there exist a version of the conditional distribution that depends continuously (in supremum norm) on the observation process. Here we tell the rest of the story and discover that in the case of dependent signal and observation, where the latter is multidimensional, there exists a version that is continuous in rough path metric. This is joint work with Dan Crisan (Imperial College London), Peter Friz (TU Berlin) and Harald Oberhauser (TU Berlin). No prior knowledge of rough path theory will be essential to follow the talk.

**Benjamin Gess (U Bielefeld).** *(Analytically) Strong Solutions for Stochastic Partial Differential Equations of Gradient Type*

We prove the unique existence of analytically strong solutions to SPDE with drift given by the subdifferential of a quasi-convex function and with general multiplicative noise. The proof applies a new method of weighted Galerkin approximations based on the “distance” defined by the quasi-convex function. Spatial regularization of the initial condition analogous to the deterministic case is obtained. The results yield a unified framework which is applied to stochastic generalized porous media equations, stochastic generalized reaction diffusion equations and stochastic generalized degenerated  $p$ -Laplace equations. In particular, higher regularity for solutions of such SPDE is obtained.

**Dennis Hagedorn (U Bielefeld).** *Dynamics related to the Gamma measure*

We use the Gamma measure in an infinite dimensional setting. To that end we outline the framework to treat infinite particle systems, i.e. the configuration spaces setting. A quasi-invariance property of the Gamma measure that leads to an integration by parts rule is given. We also present the proper gradient to apply the Dirichlet form approach.

**Michael Högele (U Potsdam).** *Metastability of stochastic reaction-diffusion equations with small regularly varying Lévy noise*

Motivated by climatological studies researchers around Imkeller and Pavlyukevich showed in a series of articles that the study of diffusions in multi-well potentials with Lévy noise with (heavy-tailed) regularly varying tails at small intensity may provide substantial conceptual insight in the behavior of paleoclimatic time series. I will advertise a natural spatial extension of this kind of models, which leads to respective parabolic stochastic partial differential equations. The talk will be devoted to the discussion of the asymptotic first exit time from the deterministic domain of attraction of a stable state in the limit of small noise intensity. The proof method is based on a path-wise approach and relies crucially on the strong Markov property of the solution and a detailed knowledge of the deterministic dynamics. We show that the expected exit time grows polynomially in the inverse noise intensity, which contrasts sharply with the exponential scales usually obtained by the Freidlin-Wentzell theory. In the end, the announced metastability result will be presented. This work is in collaboration with Peter Imkeller and Arnaud Debussche.

**Hilmar Mai (HU Berlin).** *Efficient estimation for SDEs with jumps from discrete observations*

In this talk we consider the problem of estimating the drift coefficient of a Lévy-driven Ornstein-Uhlenbeck process from discrete observations. Our method is based on discretization of the continuous-time maximum likelihood estimator. Since the likelihood function is a functional of the unknown continuous part of the process, we use a truncation method to filter jumps from the data. Then, we prove that under suitable conditions on the discretization scheme and the jump part of the process the discretized MLE with jump filtering is asymptotically normal and efficient in the sense of Hájek-Le Cam.

Finally, we discuss a simulation study to assess the finite sample behavior of the estimator and demonstrate its practical tractability.

**Harald Oberhauser (TU Berlin).** *Rough paths and SPDEs*

**Shun-Xiang Ouyang (U Bielefeld).** *Non-time-homogeneous Skew Convolution Equations*

Let  $\mathbb{H}$  be a real separable Hilbert space and  $(U(t, s))_{t \geq s}$  an evolution family of bounded operators on  $\mathbb{H}$ . Non-time-homogeneous skew convolution equations is convolution equations for probability measures  $(\mu_{t,s})_{t \geq s}$  on  $(\mathbb{H}, \mathcal{B}(\mathbb{H}))$  satisfying

$$\mu_{t,s} = \mu_{t,r} * (\mu_{r,s} \circ U(t, r)^{-1})$$

for  $t \geq r \geq s$ . In this talk we shall show the background and the continuity, infinite divisibility, construction and spectral representation of  $\mu_{t,s}$  under certain natural assumptions. This is a part of an paper on non-time-homogeneous generalized Mehler semigroup with M. Röckner.

**Nicolas Perkowski (HU Berlin).** *A Homogenized Particle Filter*

Consider a multiscale diffusion

$$\begin{aligned} dX_t^\epsilon &= b(X_t^\epsilon, Z_t^\epsilon)dt + \frac{1}{\epsilon}b_1(X_t^\epsilon, Z_t^\epsilon)dt + \sigma(X_t^\epsilon, Z_t^\epsilon)dV_t \\ dZ_t^\epsilon &= \frac{1}{\epsilon^2}f(X_t^\epsilon, Z_t^\epsilon)dt + \frac{1}{\epsilon}g(X_t^\epsilon, Z_t^\epsilon)dW_t \end{aligned}$$

where we assume that the fast component  $Z^\epsilon$  has a unique ergodic measure. Assume there is an observation

$$Y_t^\epsilon = \int_0^t h(X_s^\epsilon, Z_s^\epsilon)ds + B_t$$

We are interested in the slow component  $X^\epsilon$ , and we want to filter out the noise  $B$ . That is, we want to calculate  $\mathbb{P}(X_t^\epsilon \in \cdot | Y_s^\epsilon : s \leq t)$ . In this talk, I will present an efficient method of doing so: By combining particle filtering methods with heterogeneous multiscale methods, we are able to obtain a fast numerical algorithm to calculate an approximate filter - the homogenized hybrid particle filter. I will prove convergence of this filter as the number of particles tends to infinity and as  $\epsilon$  tends to 0. This is joint work with Sri Namachchivaya, Jun Park and Hoong Chieh Yeong (University of Illinois at Urbana-Champaign).

**Sebastian Riedel (TU Berlin).** *Applications of rough paths theory in numerical schemes*

We show how to use rough path analysis in order to obtain optimal rates of convergence for the multidimensional Wong-Zakai theorem. With the same idea, we derive a quantitative version of a well-known limit theorem for stochastic flows. We also give some ideas how to extend this result if one substitutes the driving Brownian motion by a general Gaussian process.

**Julia Ruscher (TU Berlin).** *Brownian motion with variable drift can have isolated zeros*

It is well known that a standard one-dimensional Brownian motion  $B$  has no isolated zeros almost surely. We will show that there are continuous functions  $f$  such that  $B - f$  has isolated zeros with positive probability. Furthermore, for any function  $f$ , the zero set of  $B - f$  has Hausdorff dimension of at least  $1/2$  with positive probability. This is a joint work with Tonci Antunovic, Krzysztof Burdzy and Yuval Peres.

**Adrian Schnitzler (TU Berlin).** *The parabolic Anderson model between quenched and annealed behaviour*

We consider the solution to the parabolic Anderson model, i.e. the heat equation on the lattice, with homogeneous initial condition in large time-dependent boxes. We derive stable limit theorems ranging over all possible scaling parameters for the rescaled sum over the solution depending on the growth rate of the boxes.

**Matthias Stephan (U Bielefeld).** *Yosida Approximations for Multivalued Stochastic Differential Equations on Banach spaces*

We consider multivalued stochastic differential equations with maximal monotone drift on a Gelfand triple. We prove existence and uniqueness of such equations using the Yosida approximation approach. We establish weak convergence of solutions of the approximating equations where the maximal monotone operator is replaced by its Yosida approximation. As an application the singular porous medium equation is considered.

**Ludwig Streit (U Bielefeld).** *Weakly Self Avoiding Fractional Brownian Motion - the Edwards model for collapsed polymers*

We extend Varadhan's construction of the Edwards polymer model to the case of fractional Brownian motions in  $R^d$ , for any dimension  $d \geq 2$ , with arbitrary Hurst parameters  $H \leq 1/d$ .

**Jonas Tölle (TU Berlin).** *On stochastic evolution variational inequalities*

We present an existence and uniqueness result for stochastic evolution variational inequalities modeling multi-valued stochastic evolution inclusions with additive noise and subpotential-type drift such that the convex potential satisfies a growth condition and has compact sublevel sets. Examples include the 1-Laplace and singular fast diffusion equations. Another application is given by continuous dependence of the solutions on the potential with respect to so-called Mosco-convergence of convex functionals. We also discuss existence of invariant measures.

**Jörg Vorbrink (U Bielefeld).** *Financial markets with volatility uncertainty*

We investigate financial markets under model risk caused by uncertain volatilities. For this purpose we consider a financial market that features volatility uncertainty. To have a mathematical consistent framework we use the notion of G-expectation and its corresponding G-Brownian motion recently introduced by Peng (2007). Our financial market consists of a riskless asset and a risky stock with price process modeled by a geometric G-Brownian motion. We adapt the notion of arbitrage to this more complex situation and consider stock price dynamics which exclude arbitrage opportunities. Due to volatility uncertainty

the market is not complete any more. We establish the interval of no-arbitrage prices for general European contingent claims and deduce explicit results in a Markovian setting.

**Tilman Wolff (WIAS Berlin).** *Random Walk among Random Conductances*

We consider a certain model of a random walk in random conductances. Understanding a RWRC involves studying certain spectral properties of a randomly perturbed discrete Laplace operator. The functional analytic approach combines with large deviation theory in order to derive a variational formula that describes the expected long-time behaviour of certain occupation time measures.

**Jianing Zhang (HU Berlin).**  *$L^p$ -solutions of BSDEs with time delayed generators*

We deal with the theory of BSDEs with time delayed generators introduced by Delong and Imkeller (2010). These BSDEs are characterized by generators which typically depend on the past of the value and the control component up to current time. Assuming Lipschitz assumptions on these path dependent generators, Delong and Imkeller establish an  $L^2$  space characterization of BSDE solutions. We amend this with an  $L^p$  space theory and discuss the various obstacles which had to be overcome. We then apply our results to study the case of a time delayed backward equation with a Markovian terminal condition and derive representation formulas for the BSDE solution which are analogous to those for BSDEs with standard (non-time delayed) generators. This is a joint work with Gonalo dos Reis (TU Berlin) and Anthony R veillac (HU Berlin).

**Rongchan Zhu (U Bielefeld).** *BSDE and generalized Dirichlet forms*

We consider the following quasi-linear parabolic system of backward partial differential equations

$$(\partial_t + L)u + f(\cdot, \cdot, u, \nabla u \sigma) = 0 \text{ on } [0, T] \times \mathbb{R}^d \quad u_T = \phi,$$

where  $L$  is a second order differential operator with measurable coefficients. We solve this system in the framework of generalized Dirichlet forms and employ the stochastic calculus associated to the Markov process with generator  $L$  to obtain a probabilistic representation of the solution  $u$  by solving the corresponding BSDE. The solution satisfies the mild equation which is equivalent to the generalized solution of the PDE. We generalize the martingale representation theorem using the stochastic calculus associated to the generalized Dirichlet form. The nonlinear term  $f$  satisfies a monotonicity condition with respect to  $u$  and a Lipschitz condition with respect to  $\nabla u$ .

**Xiangchan Zhu (U Bielefeld).** *BV functions in a Gelfand triple and the stochastic reflection problem on a convex set of a Hilbert space*

In this paper, we introduce a definition of BV functions in a Gelfand triple which is an extension of the definition of BV functions in [1] by using Dirichlet form theory. By this definition, we can consider the stochastic reflection problem associated with a self-adjoint operator  $A$  and a cylindrical Wiener process on a convex set  $\Gamma$  in a Hilbert space  $H$ . We prove the existence and uniqueness of a strong solution of this problem when  $\Gamma$  is a regular convex set. The result is also extended to the non-symmetric case. Finally, we extend our results to the case when  $\Gamma = K_\alpha$ , where  $K_\alpha = \{f \in L^2(0, 1) | f \geq -\alpha\}, \alpha \geq 0$ .