Junior female researchers in probability

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Random walks in a 1D Levy random environment

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joint work with G.Cristadoro, M. Lenci, M. Ligabó

University of Bologna

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Outline

1. MOTIVATIONS
   Anomalous diffusions

2. RELATED MODELS AND RESULTS
   Levy flights and walks and annealed results

3. MODEL
   1D Random walk in Levy random environment

4. RESULTS AND SOME IDEAS OF THE PROOF
   Quenched distribution and moments

5. CONCLUSIONS, WORKS IN PROGRESS AND OPEN PROBLEMS

Anomalous diffusions

Anomalous diffusions are stochastic processes $X(t) \in \mathbb{R}^d$ such that

$$\mathbb{E}(X^2(t)) \sim t^\delta \quad \text{for } t \to \infty, \quad \delta \neq 1$$

The behavior of superdiffusive processes ($\delta > 1$) characterizes many different natural systems and is mainly connected to
motion in disorder media:

- light particle in an optical lattice;
- tracer in a turbolent flow;
- efficient routing in network;
- predator hunting for food

Main features

- long ballistic "flights", where particle moves at constant velocity
- short disorder motion

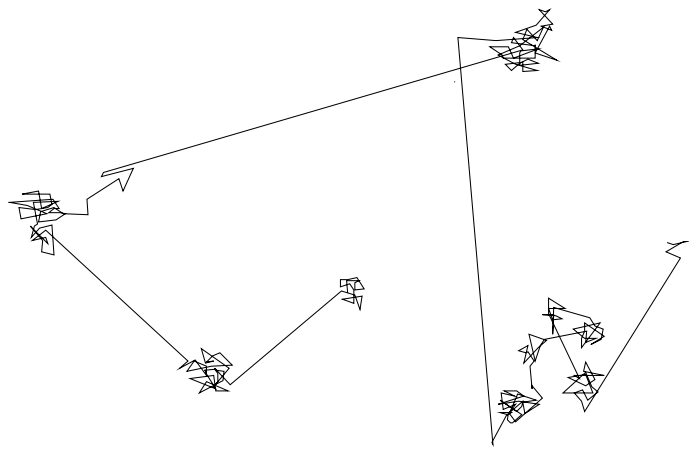

Figure 1: Typical Levy flight

Models for anomalous diffusions

LEVY FLIGHTS

Schlesinger, Klafter['85] , Blumen, Klafter, Schlesinger, Zumofen ['90] ,

Random walk $(X(n))_{n\in\mathbb{N}}$ on $\mathbb{R}^d$ with lenght steps given by a sequence of i.i.d. Levy α-stable distribution with $\alpha \in (0,2)$:

heavy-tailed distribution $\quad \mathbb{P}(Z > x) \sim x^{-\alpha}$ for $x \to +\infty$

$$\longrightarrow \quad \mathrm{Var}(Z) = +\infty \quad ; \quad \mathbb{E}(Z) \begin{cases} < \infty & \text{if } \alpha \in (1,2) \\ = \infty & \text{if } \alpha \in (0,1] \end{cases}$$

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Formally:

Given $(\xi_k)_{k\in\mathbb{N}}$, i.i.d. U$[S^{d-1}]$, independent of $(Z_k)_{k\in\mathbb{N}}$, i.i.d Levy α-stable

$$X(0) = 0 \quad , \quad X(n) = X(n-1) + Z_n \xi_n, \quad n \geq 0$$

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Stochastic process $(X(t))_{t\in\mathbb{R}^+}$ on $\mathbb{R}^d$ defined similarly to Levy flights but with jumps covered at constant velocity v_0.

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Notice: in both processes **the lengths of the jumps (or inter-collision times) are independent**,

$$\Downarrow \qquad \Downarrow$$

scatterers are removed after each collision event.

Results on the second moments

Levy flights and walks give rise to superdiffusive anomalous motion and in particular

$$\mathbb{E}(X^2(t)) \sim \begin{cases} t^2 & \text{if } \alpha \in (0,1] \\ t^{3-\alpha} & \text{if } \alpha \in (1,2) \end{cases} \qquad \text{for } t \to \infty$$

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LEVY-LORENTZ GAS
Barkai, Fleurov,Klafter['00]

Motion of a particle in a fixed array of scatterers arranged randomly in such a way that the interdistances between them are i.i.d. α-stable Levy random variables.

MODEL

Levy Random environment

• Let $(Z_k)_{k\in\mathbb{Z}}$ i.i.d. random variables taking value on $\mathbb{N}^+$ and with law P s.t.

$$P(Z > k) \sim k^{-\alpha} \quad \text{for } k \gg 1 \qquad \text{(heavy tails)}$$

• Construct a Renewal Point Process on $\mathbb{Z}$, denoted by
PP$(Z) = \{\ldots Y_{-1} < Y_0 < Y_1 < \ldots\}$, s.t.

1. $Y_0 = 0$
2. $|Y_k - Y_{k-1}| = Z_k$ so that $Y_k = \text{sgn}(k) \sum_{j=1}^{|k|} Z_{\text{sgn}(k)j}\,, \quad k \neq 0$

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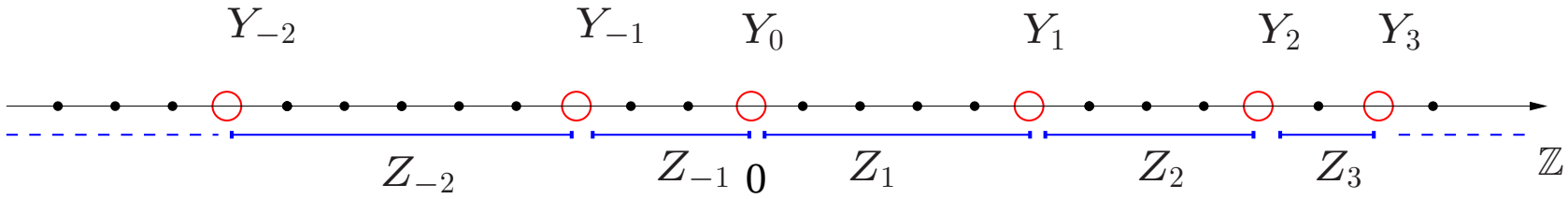

1D random walk in a Levy Random environment

- Let $(\xi_k)_{k\in\mathbb{N}}$ i.i.d. symmetric random variables taking value on $\{-1,+1\}$.

Definition 1. $X(t)$, $t \in \mathbb{N}$ is the process on $\mathbb{Z}$ such that

$X(0) = 0$
$X(t+1) = X(t) + \xi_{n(t)}$, for $t > 0$

with $n(t) = |\{s \leq t \,:\, X(s) \in \mathrm{PP}(Z)\}| =$ number of collisions up to t.

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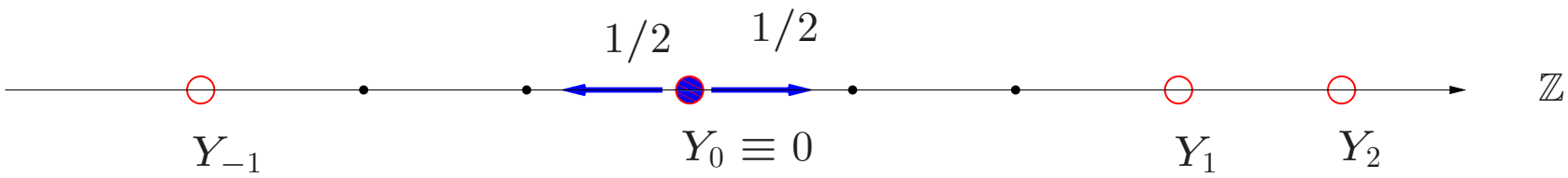

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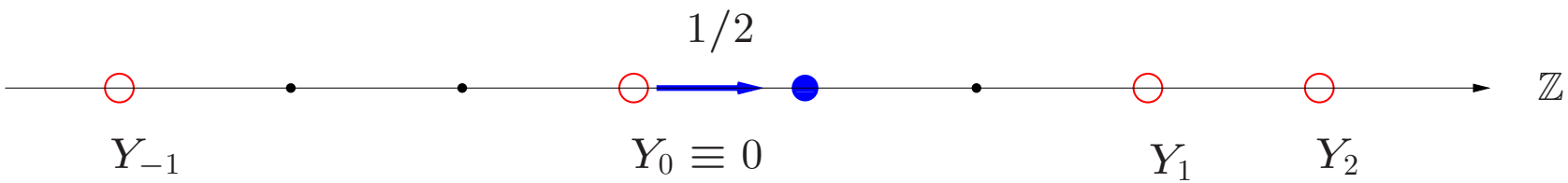

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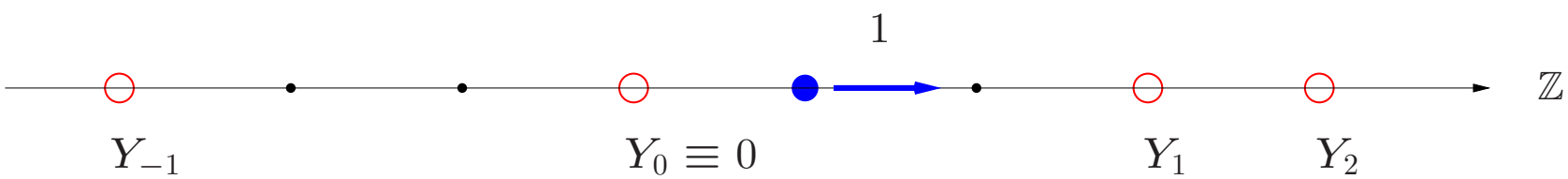

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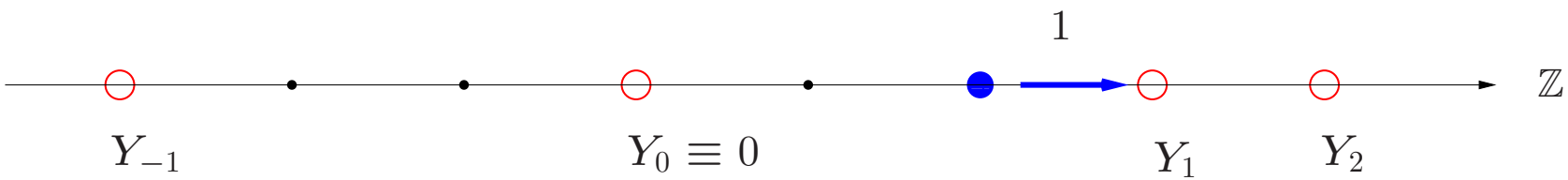

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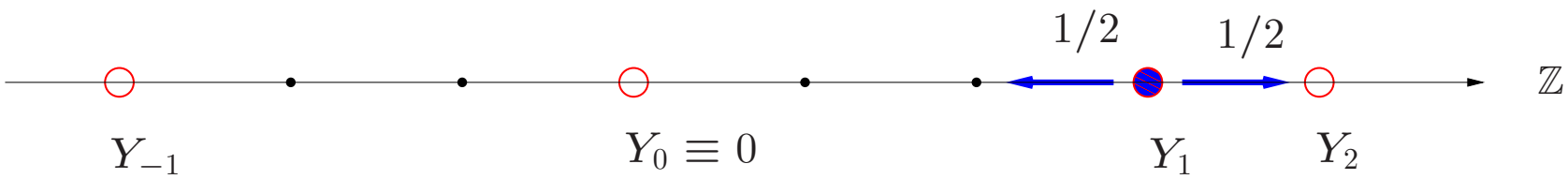

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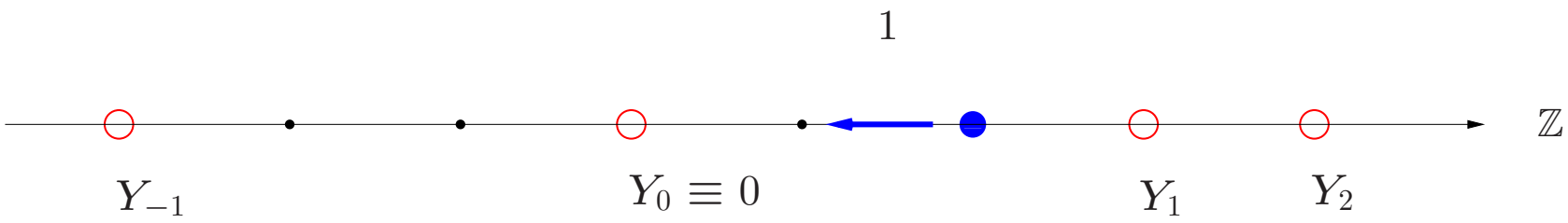

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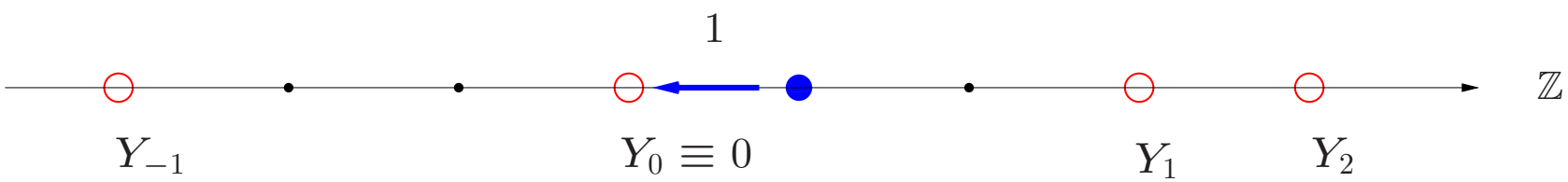

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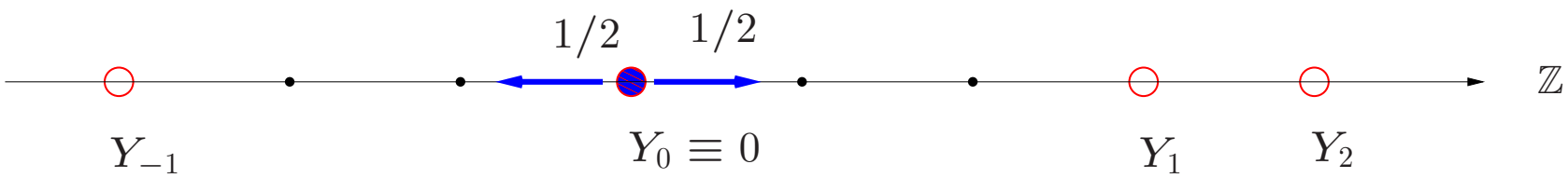

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Remark: The random environment is not i.i.d., not even stationary, not elliptic, and the variance is infinite $\longrightarrow$ standard methods do not apply.

Previous (annealed) results

• For $\alpha \geq 2$ (finite variance) studied by Grassberger ('80), van Beijeren; Spohn ('83), Ernst; Dorfman; Nix; Jacobs ('95), Barkai; Fleurov ('99):
(i) $\mathbb{E}(X^2(t)) \sim t$
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(i) $\mathbb{E}(X^2(t)) \geq c(\alpha)t^{2-\alpha}$ for PP(Z) conditioned to contain 0
(ii) $\mathbb{E}(X^2(t)) \geq c(\alpha)t^{3-\alpha}$ for stationary PP(Z)

where in stationary PP(Z), $P(Y_1 = \ell) = \frac{\ell \mathbb{P}(Z=\ell)}{\mathbb{E}(Z)}$

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Main tools: Laplace transform and Tauberian theorem.

The result is compatible with a Levy flight scheme but not much informative for non-stationary initial conditions. Nothing is known about the quenched process.

Process at collision times

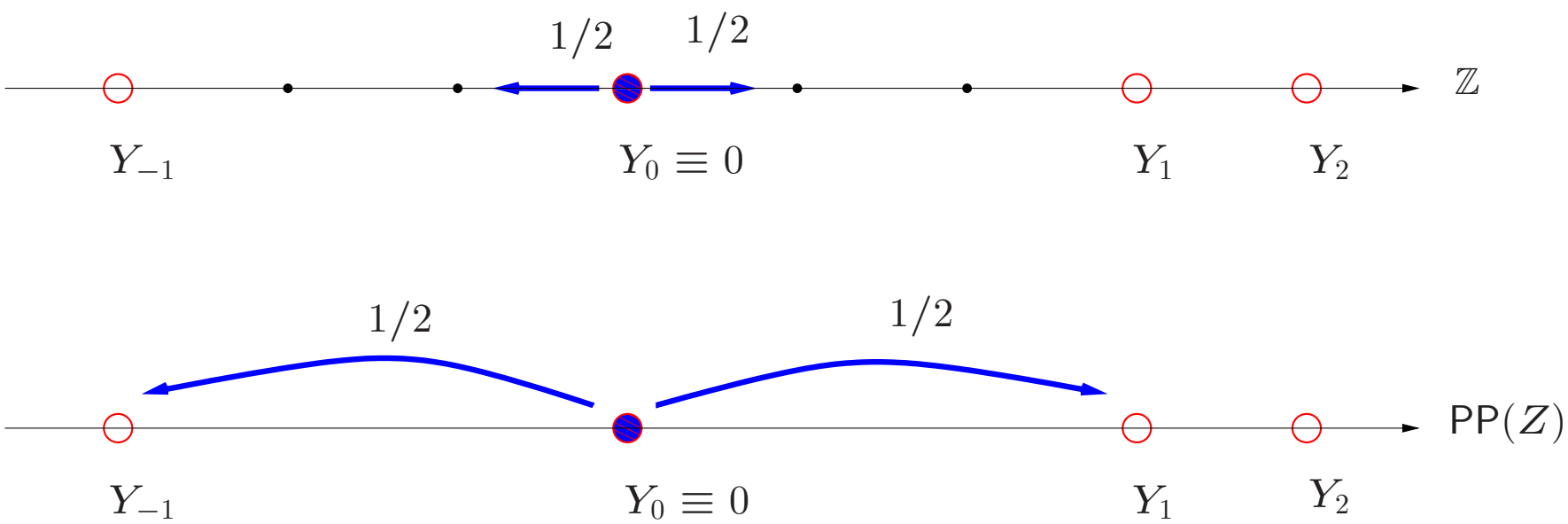
For $n \in \mathbb{N}$, let $t(n)$ =time of the nth collision
and set

$$\tilde{X}(n) \equiv X(t(n))\,, \quad n \in \mathbb{N}$$

- $\tilde{X}(n)$ is a SSRW on PP(Z).
- Letting $S_n = \sum_{k=1}^{n} \xi_k$ the coupled SSRW on $\mathbb{Z}$, it holds

$$\tilde{X}(n) = Y_{S_n}$$

that is, $\tilde{X}(n)$ is the position of scatter label by S_n.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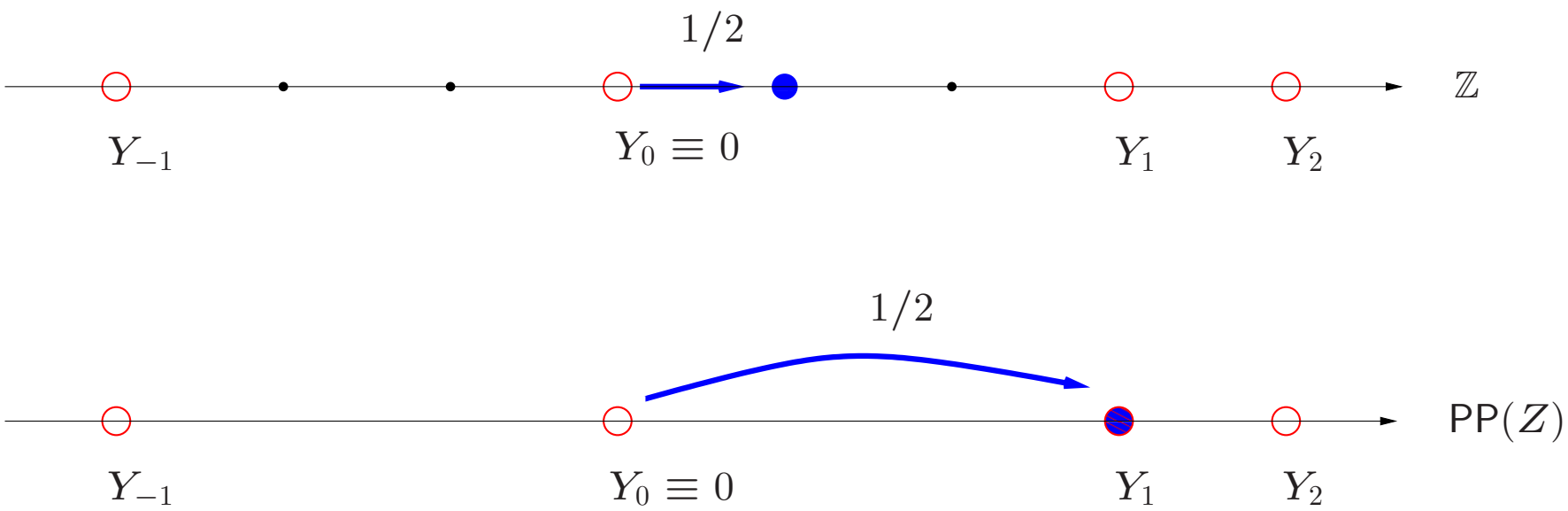
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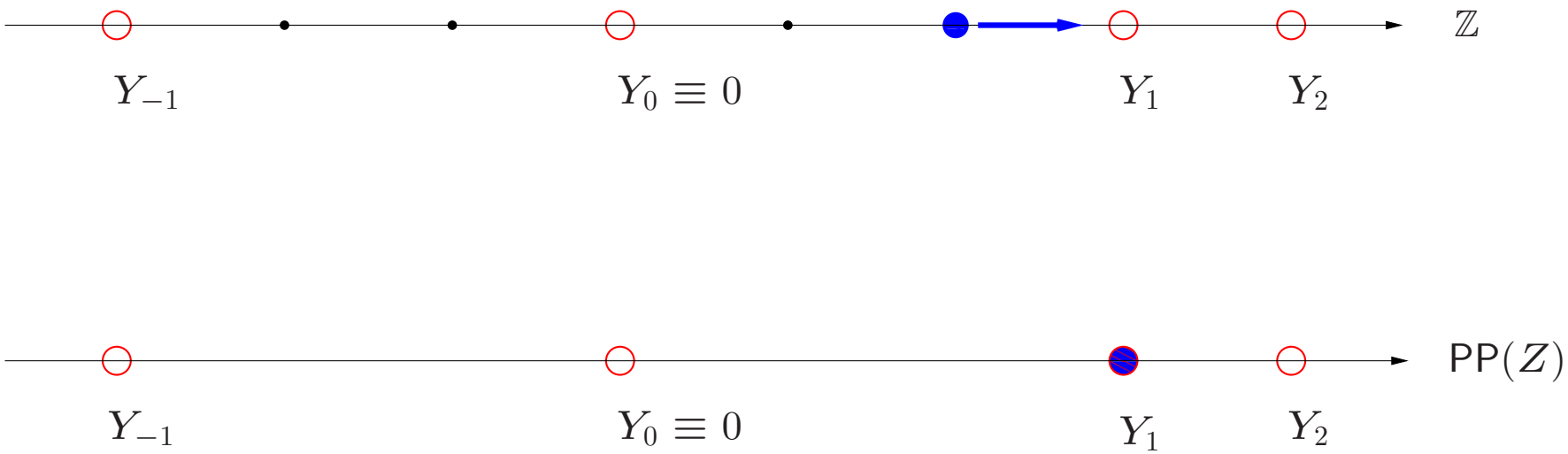
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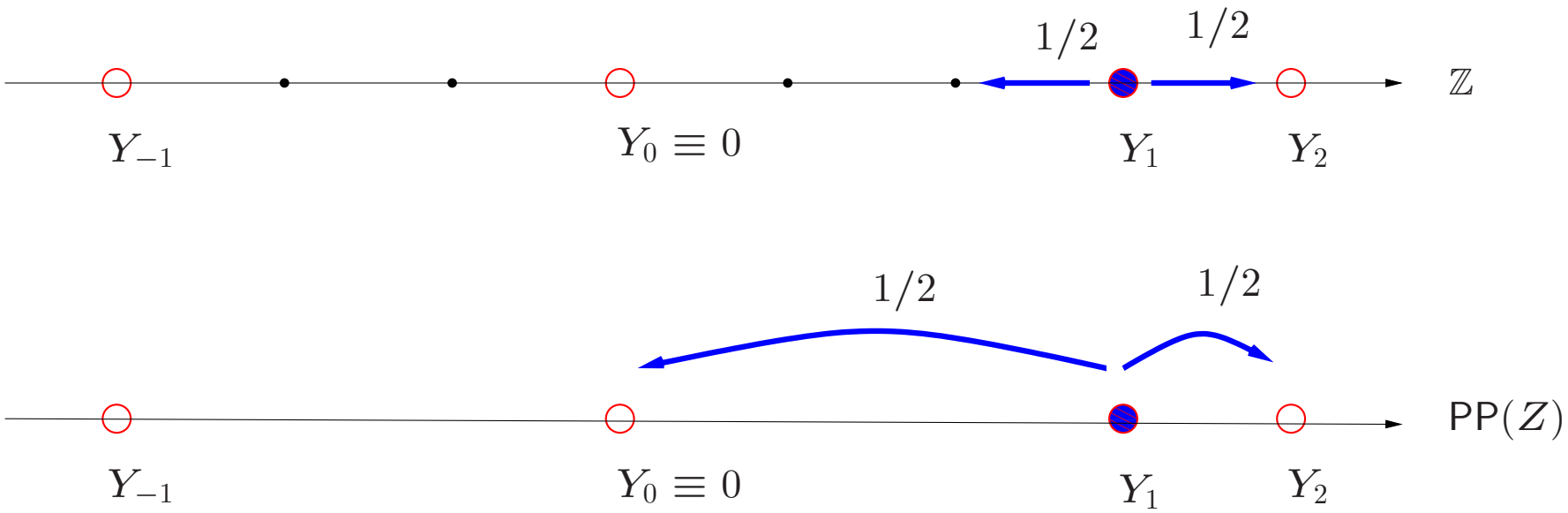
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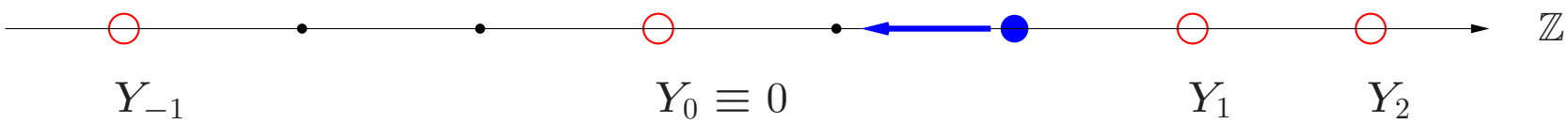
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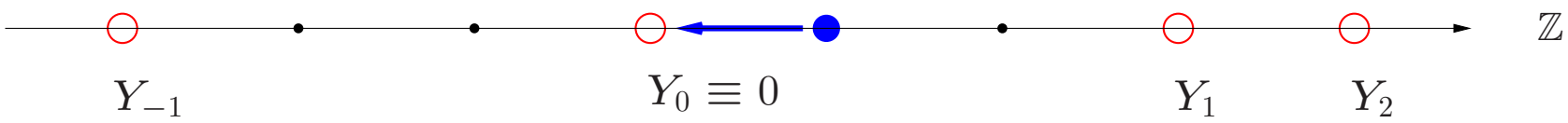
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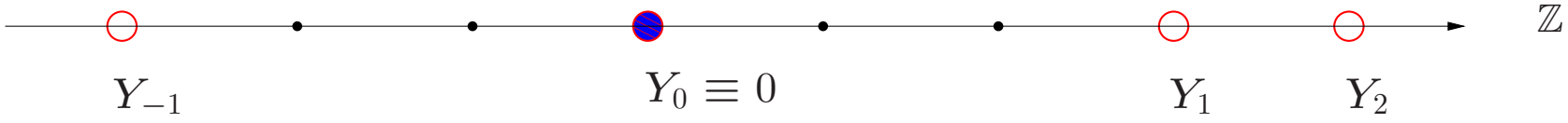
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Quenched law of $\tilde{X}(n)$

Proposition 1. *For $\alpha \in (1,2)$ and a PP(Z) conditioned to contain 0, it holds*

$$P_z\left(\frac{\tilde{X}(n)}{\mu\sqrt{n}} > x\right) \xrightarrow{n\to\infty} \int_x^{+\infty} \frac{e^{-t^2/2}}{\sqrt{2\pi}}dt \qquad P\text{-a.s.}$$

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Proof idea: From $\tilde{X}(n) = Y_{S_n}$, we used

- CLT for S_n
- LLN for Y_k

Quenched law of $X(t)$

THM 1. *For $\alpha \in (1,2)$ and a PP(Z) conditioned to contain 0, it holds*

$$P_z\left(\frac{X(t)}{\sqrt{\mu t}} > x\right) \xrightarrow{t\to\infty} \int_x^{+\infty} \frac{e^{-t^2/2}}{\sqrt{2\pi}} dt \qquad P\text{-a.s.}$$

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Proof idea: Write $\frac{X(t)}{\sqrt{\mu t}} = \frac{X(t)-\tilde{X}(n(t))}{\sqrt{\mu t}} + \frac{\tilde{X}(n(t))}{\mu\sqrt{n(t)}}\sqrt{\mu\frac{n(t)}{t}}$

- $\mathbb{E}_z\left(\left|\frac{X(t)-\tilde{X}(n(t))}{\sqrt{\mu t}}\right|\right) \xrightarrow{t\to\infty} 0\,, \quad P-\text{a.s}$
- By the ergodicity of the annealed process for the PVP

$$\frac{n(t)}{t} \xrightarrow{t\to\infty} \frac{1}{\mu}\,, \quad \mathbb{P}-\text{a.s}$$

Quenched Moments of $\tilde{X}(n)$

Proposition 2. *For $\alpha \in (1,2)$ and a PP(Z) conditioned to contain 0, it holds*

$$E_z\left(\frac{\tilde{X}^m(n)}{n^{\frac{m}{2}}}\right) \xrightarrow{n\to\infty} \begin{cases} 0 & \text{for } m = 2k-1 \\ \mu^m (m-1)!! & \text{for } m = 2k \end{cases}, \quad P\text{-a.s.}$$

i.e., to the moments of $N(0,\mu^2)$.

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Proof idea: From $\tilde{X}(n) = Y_{S_n}$, we used

- Moments convergence for S_n
- LLN for Y_k

Quenched Moments of $X(t)$

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$$E_z\left(\frac{X^m(t)}{t^{\frac{m}{2}}}\right) \xrightarrow{n\to\infty} \begin{cases} 0 & \text{for } m = 2k-1 \\ \mu^{\frac{m}{2}}(m-1)!! & \text{for } m = 2k \end{cases}, \quad P\text{-a.s.}$$

i.e., to the moments of $N(0,\mu)$.

Proof idea: Write $\frac{X^m(t)}{t^{\frac{m}{2}}} = \frac{X^m(t)-\tilde{X}^m(n(t))}{t^{\frac{m}{2}}} + \frac{\tilde{X}^m(n(t))}{n(t)^{\frac{m}{2}}}\left(\frac{n(t)}{t}\right)^{\frac{m}{2}}$

- For $\gamma > \frac{1}{2}$, define the event $E = \left\{|X(t)| \wedge |\tilde{X}(n(t))| < t^\gamma\right\}$

s.t. $P_z(E^c) \leq e^{-t^\gamma}$ P-a.s

Then

- $\mathbb{E}_z\left(\left|\frac{X^m(t)-\tilde{X}^m(n(t))}{t^{\frac{m}{2}}}\right| \middle| E^c\right) P_z(E^c) \leq 2t^{\frac{m}{2}}e^{-t^\gamma}$

- $\mathbb{E}_z\left(\left|\frac{X^m(t)-\tilde{X}^m(n(t))}{t^{\frac{m}{2}}}\right| \middle| E\right) P_z(E)$

$$\leq \mathbb{E}_z\left(|X(t) - \tilde{X}(n(t))||E\right) \cdot mt^{\gamma(m-1)-\frac{m}{2}}$$

Choosing $\frac{1}{2} < \gamma < \frac{m}{2(m-1)}$ and from $\mathbb{E}_z\left(|X(t) - \tilde{X}(n(t))|\right) \xrightarrow{t\to\infty} \mu$ we conclude.

Corollary 1. *For $\alpha \in (1,2)$ and a PP(Z) conditioned to contain 0, it holds*

$$\mathbb{E}\left(X^2(t)\right) \geq t\,, \quad \textit{for } t \gg 1$$

This improves the annealed bound on the second moment given by BFK['00].

Conclusion, work in progress, open problems

• The quenched behavior of the 1 D Levy Lorentz gas with non-stationary initial condition do not displays anomalous diffusive behavior.

• Improved bound on the annealed second moment has been provided. Its exact behavior has still to be determined (work in progress).

• Under the stationary initial condition, we expect to find a similar behavior (work in progress)

• Study the model for $\alpha \in (0, 1]$ (infinite-mean inter-collision times). A quenched super-diffusive behavior is conjecured (work in progress).

• Provide a similar construction for a 2 D Levy Lorentz gas (open problem).

Thank you for your attention!