Front progression in the East model

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October 10th 2013

[Stochastic Process. Appl. (123 – 9), 2013, p. 3430–3465]

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Continuous time Markov process on $\{0,1\}^{\mathbb{Z}}$.

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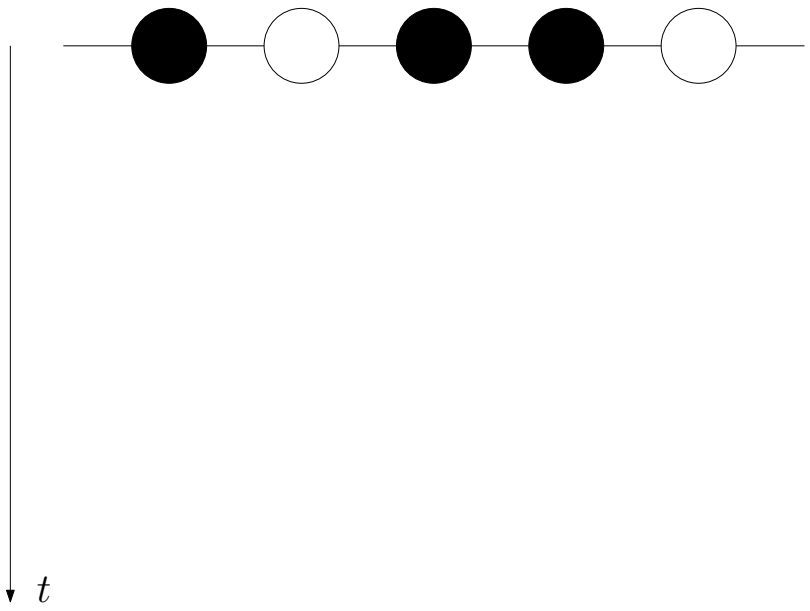
Generator

$$\mathcal{L}f(\eta) = \sum_{x\in\mathbb{Z}} (1-\eta_{x+1})(p(1-\eta_x) + (1-p)\eta_x)\left[f\left(\eta^x\right) - f(\eta)\right],$$

where

$$\eta_y^x = \begin{cases} 1-\eta_x & \text{if } y = x \\ \eta_y & \text{if } y \neq x \end{cases}$$

Graphical construction



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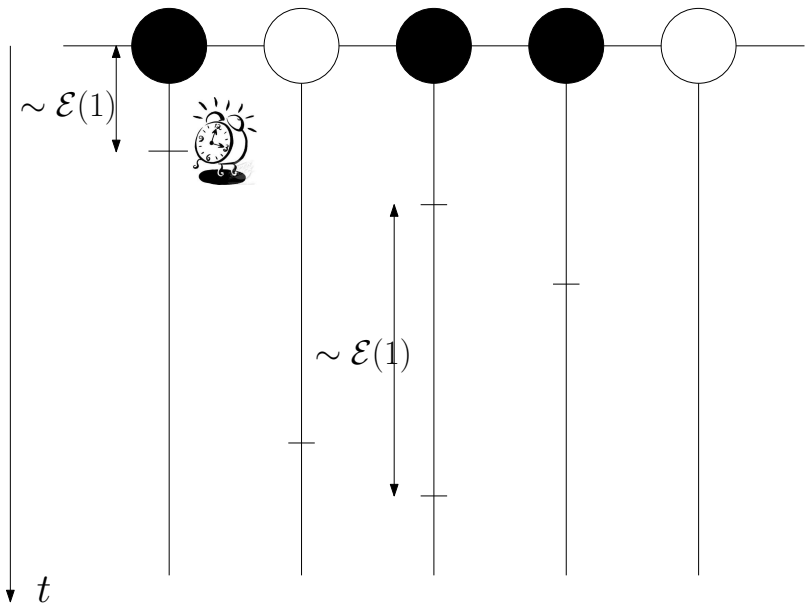

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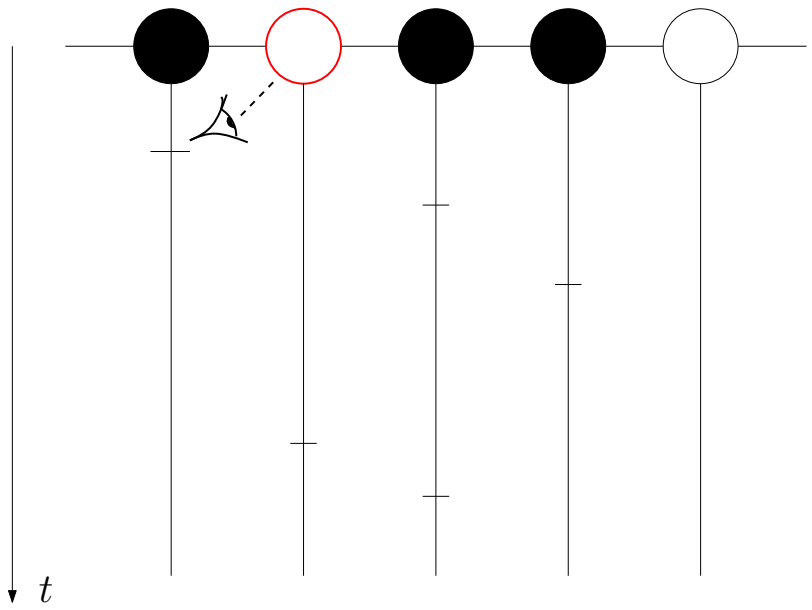

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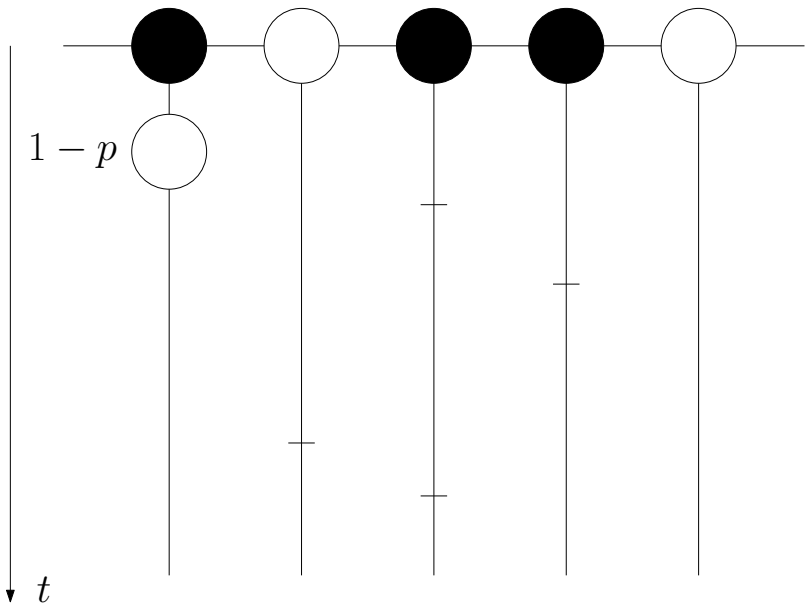

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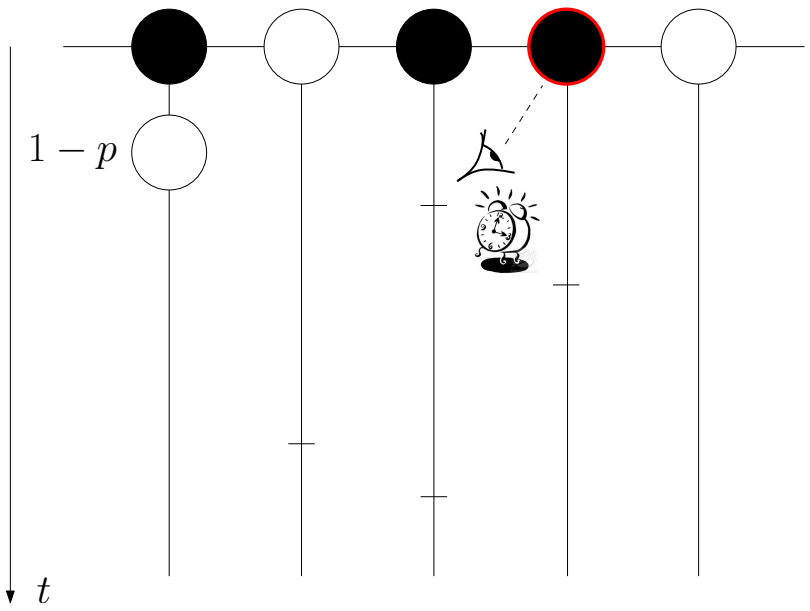

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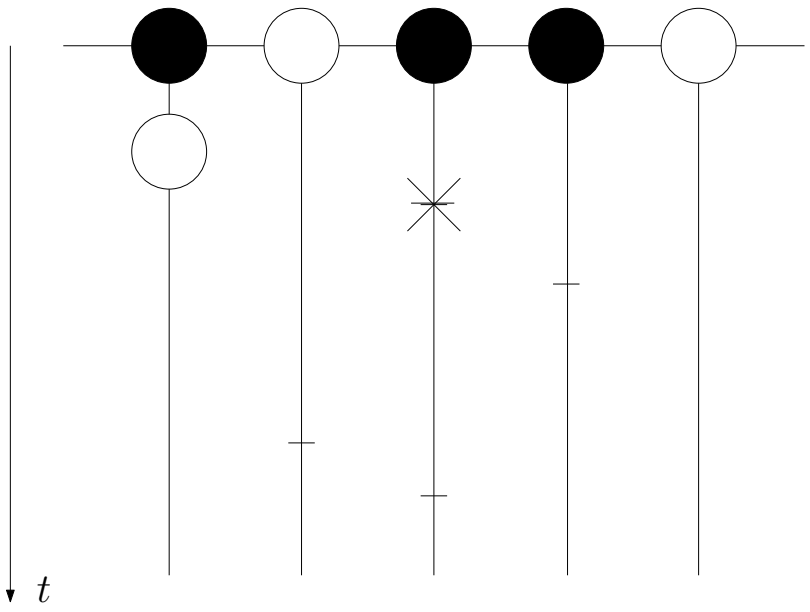

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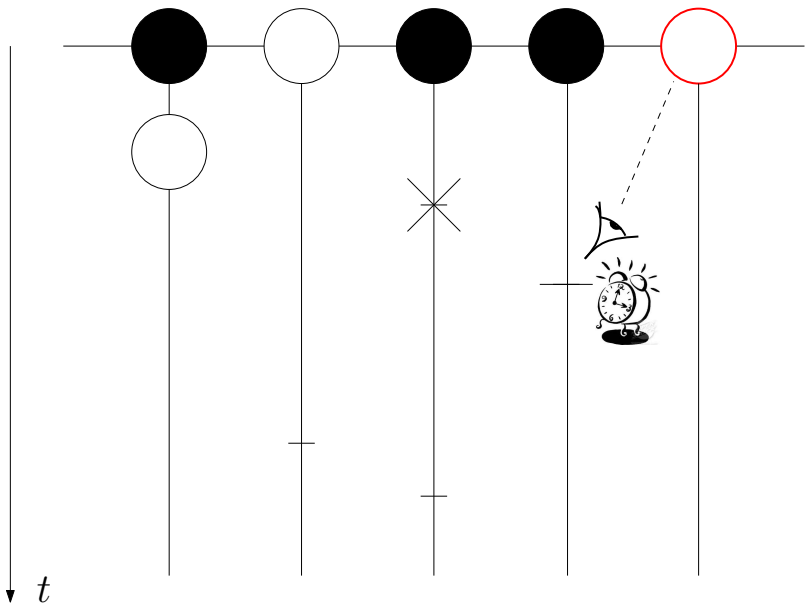

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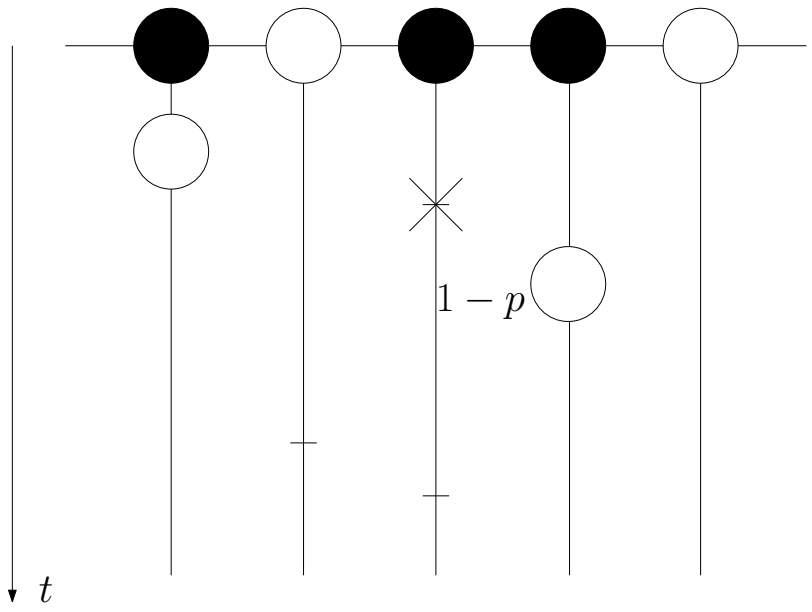

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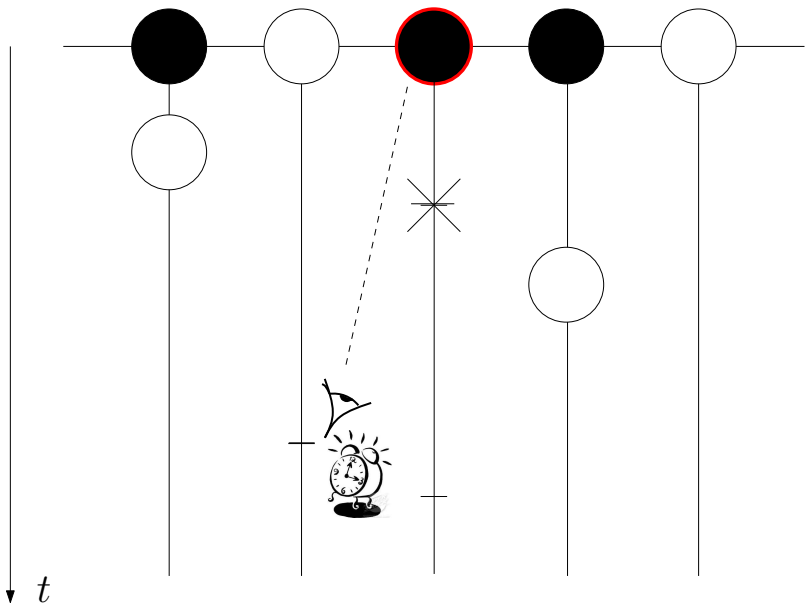

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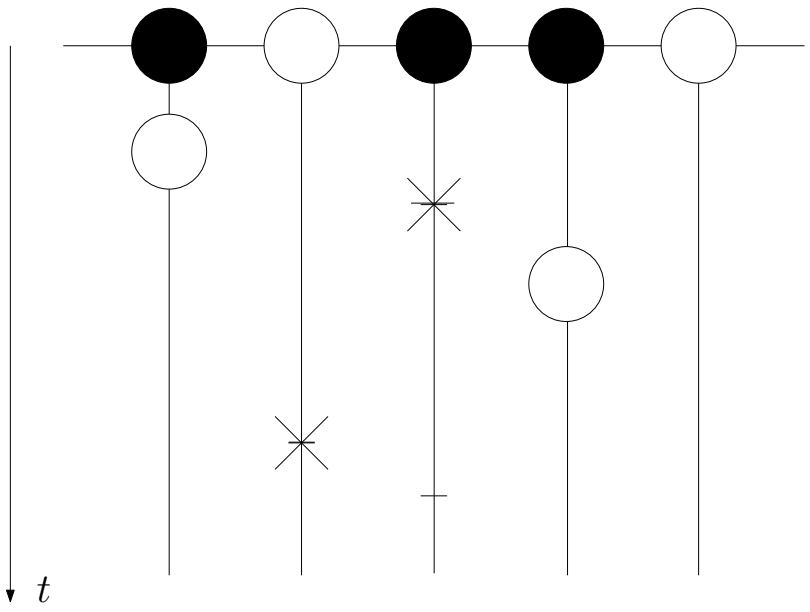

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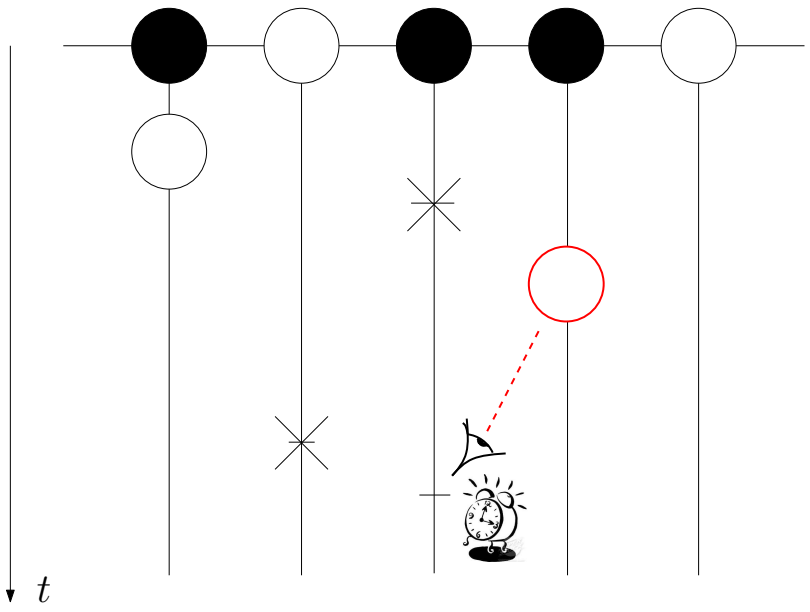

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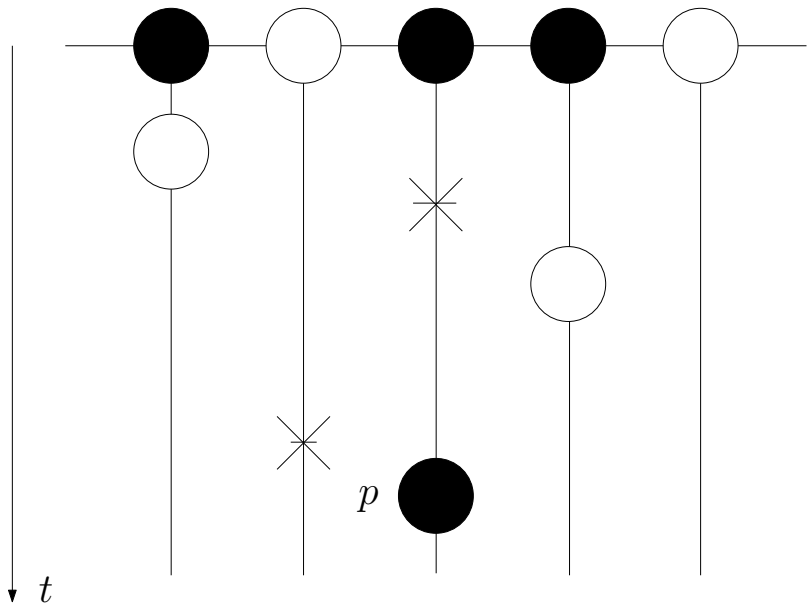

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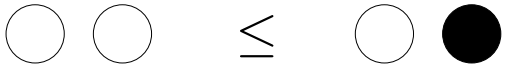

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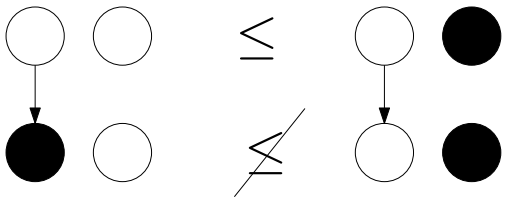

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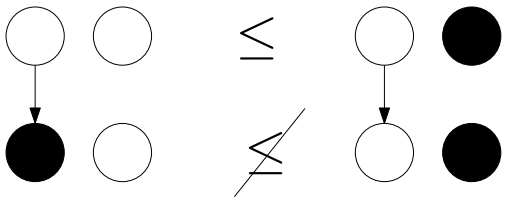

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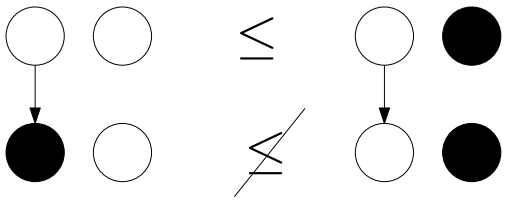
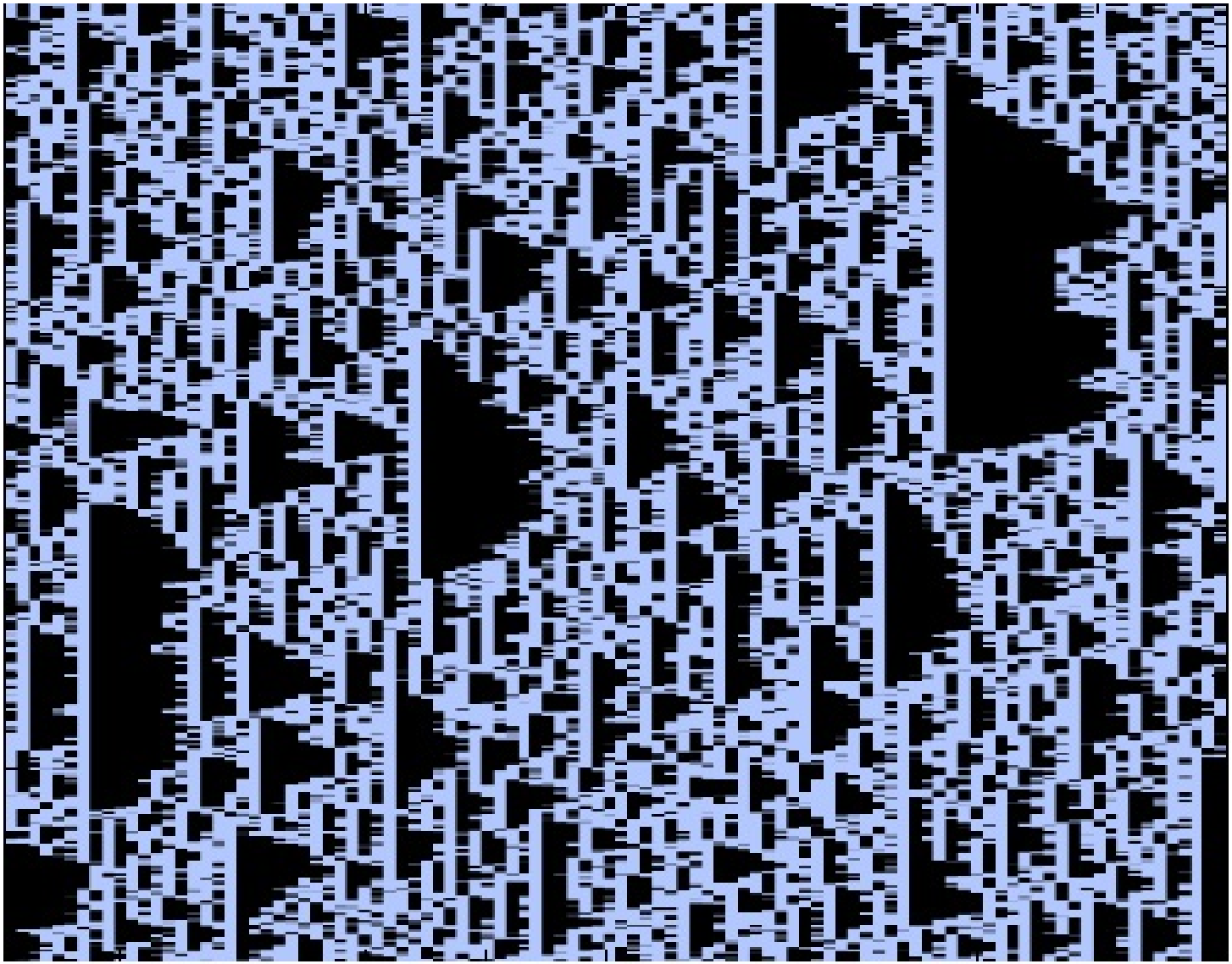
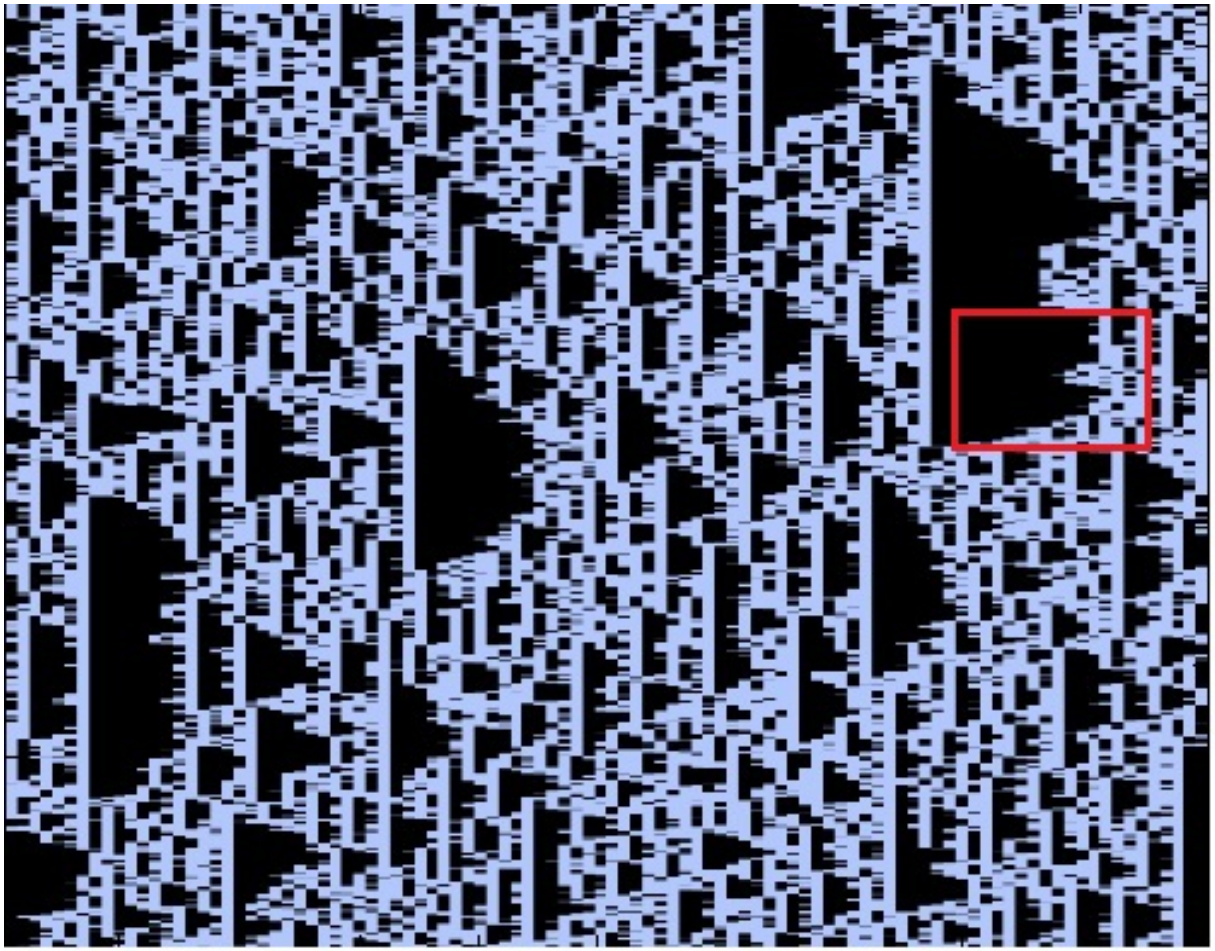
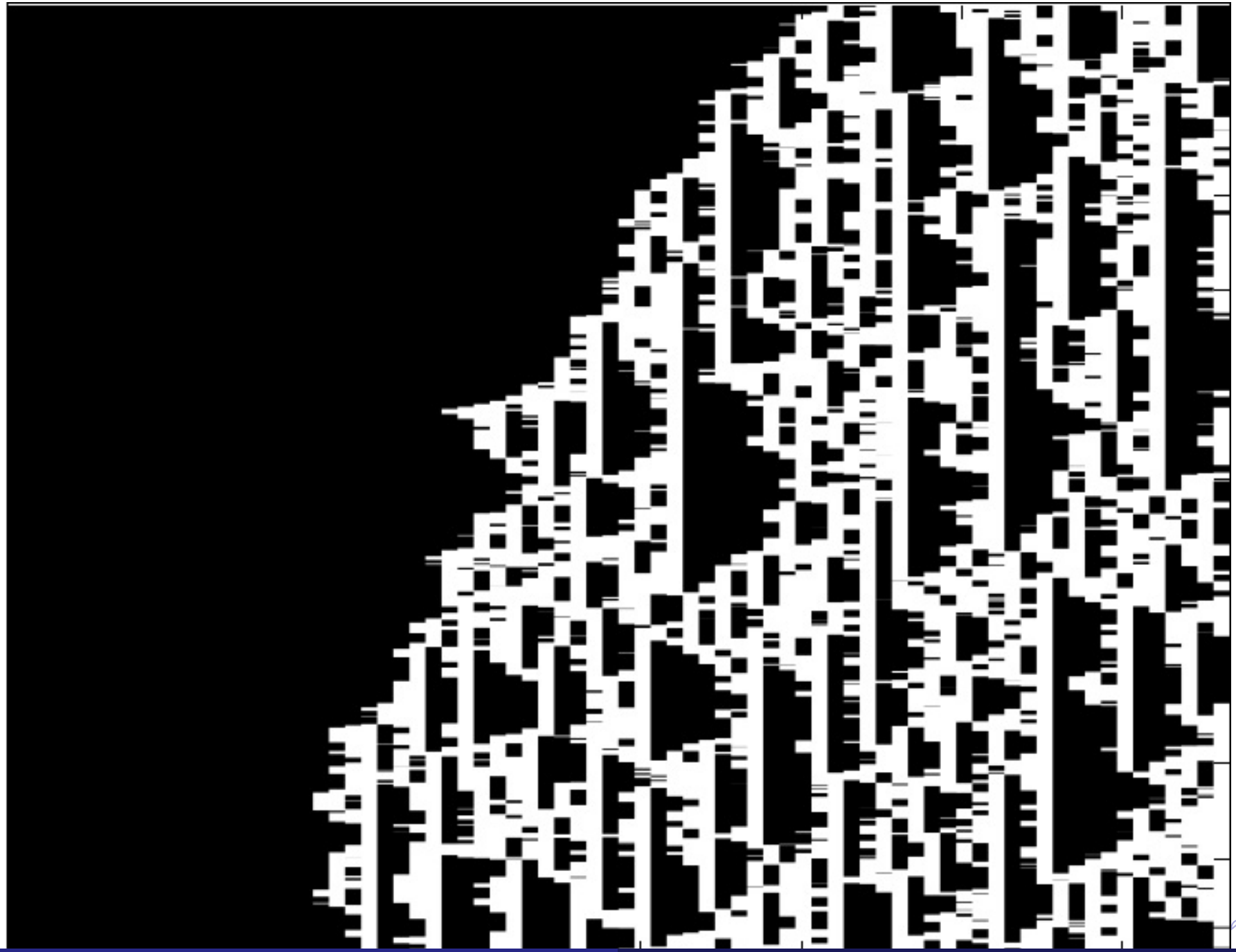


- Equilibrium measure $\mu = \mathcal{B}(p)^{\otimes \mathbb{Z}}$ (reversible)
- Exponential return to equilibrium, but not uniform.







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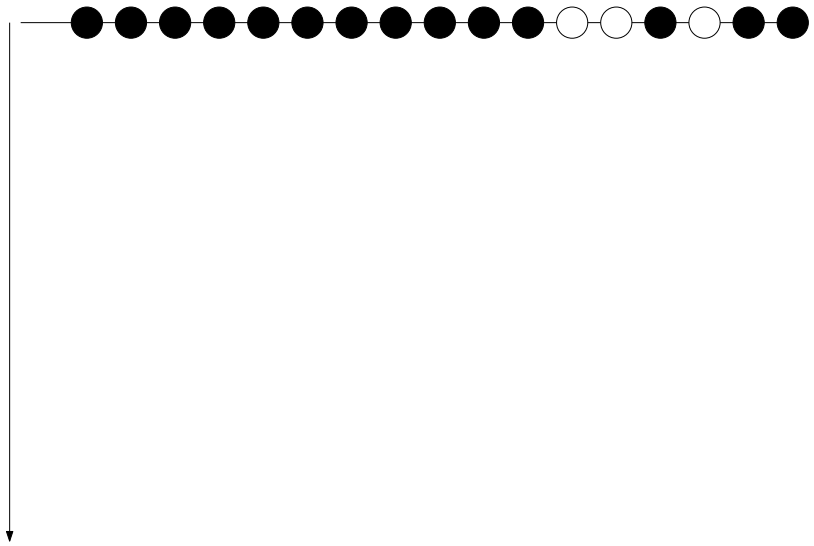

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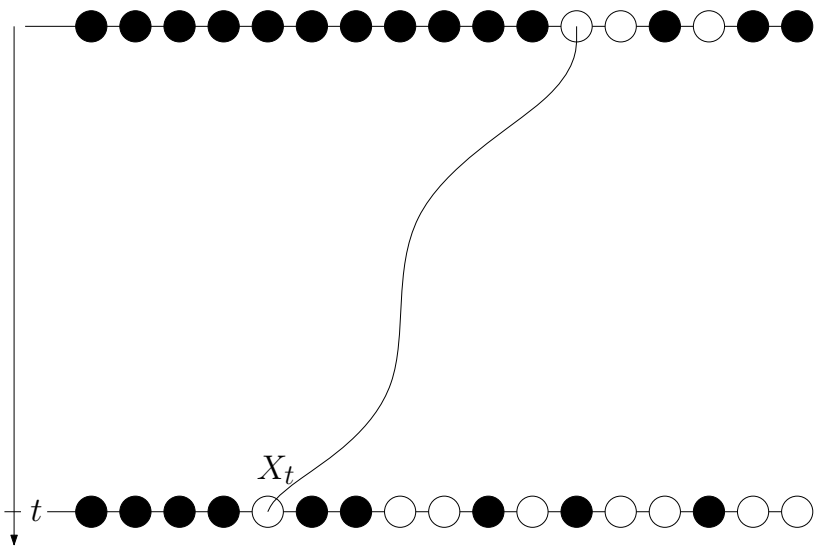

- Start from any configuration with right-most zero at 0.
- Let the East dynamics run for time t.

X_t: position of the front (*i.e.* the right-most zero) at time t.
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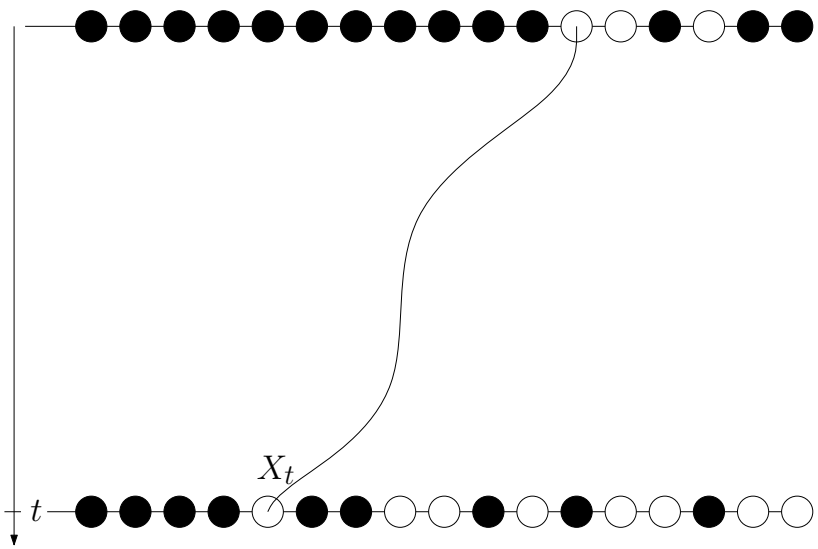

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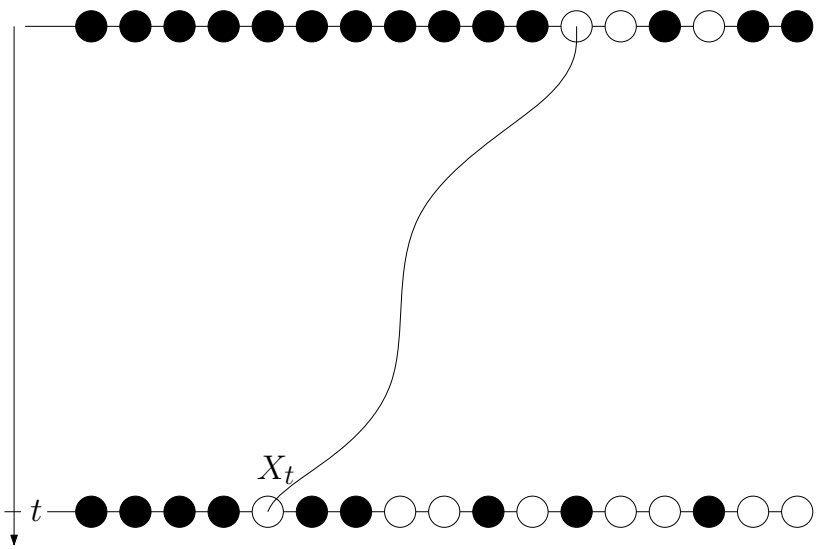

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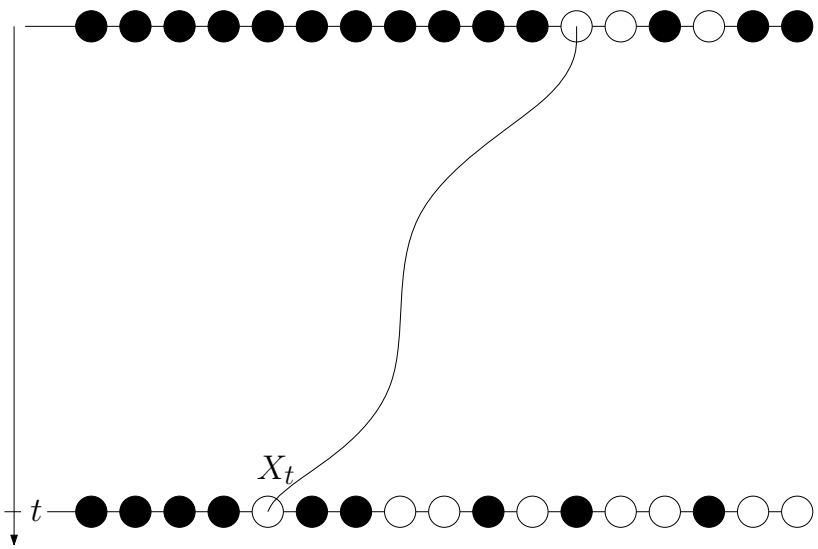

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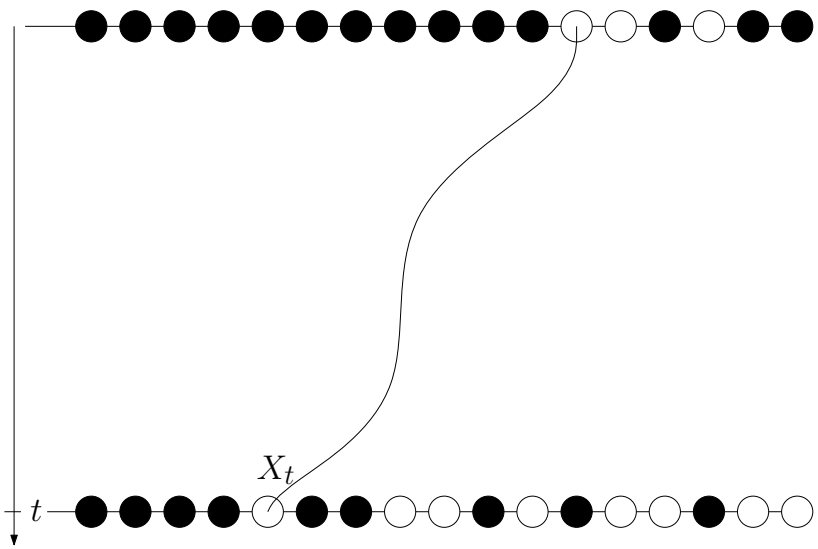

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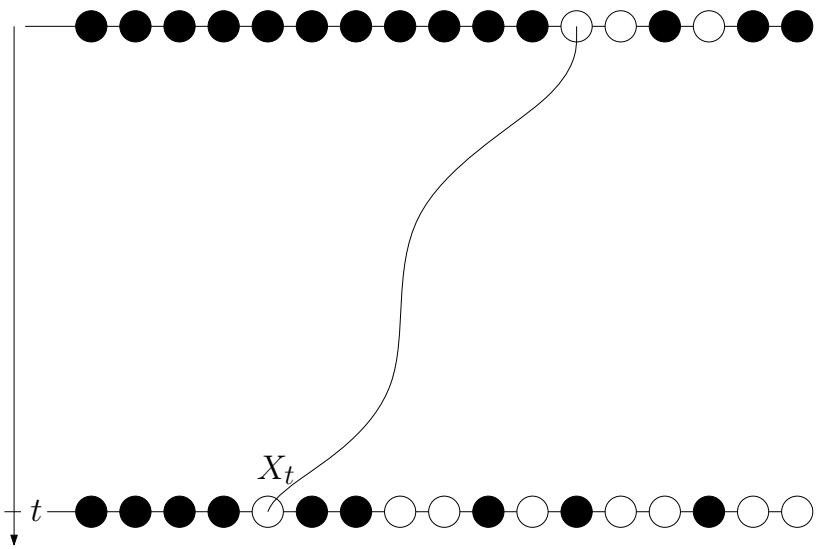

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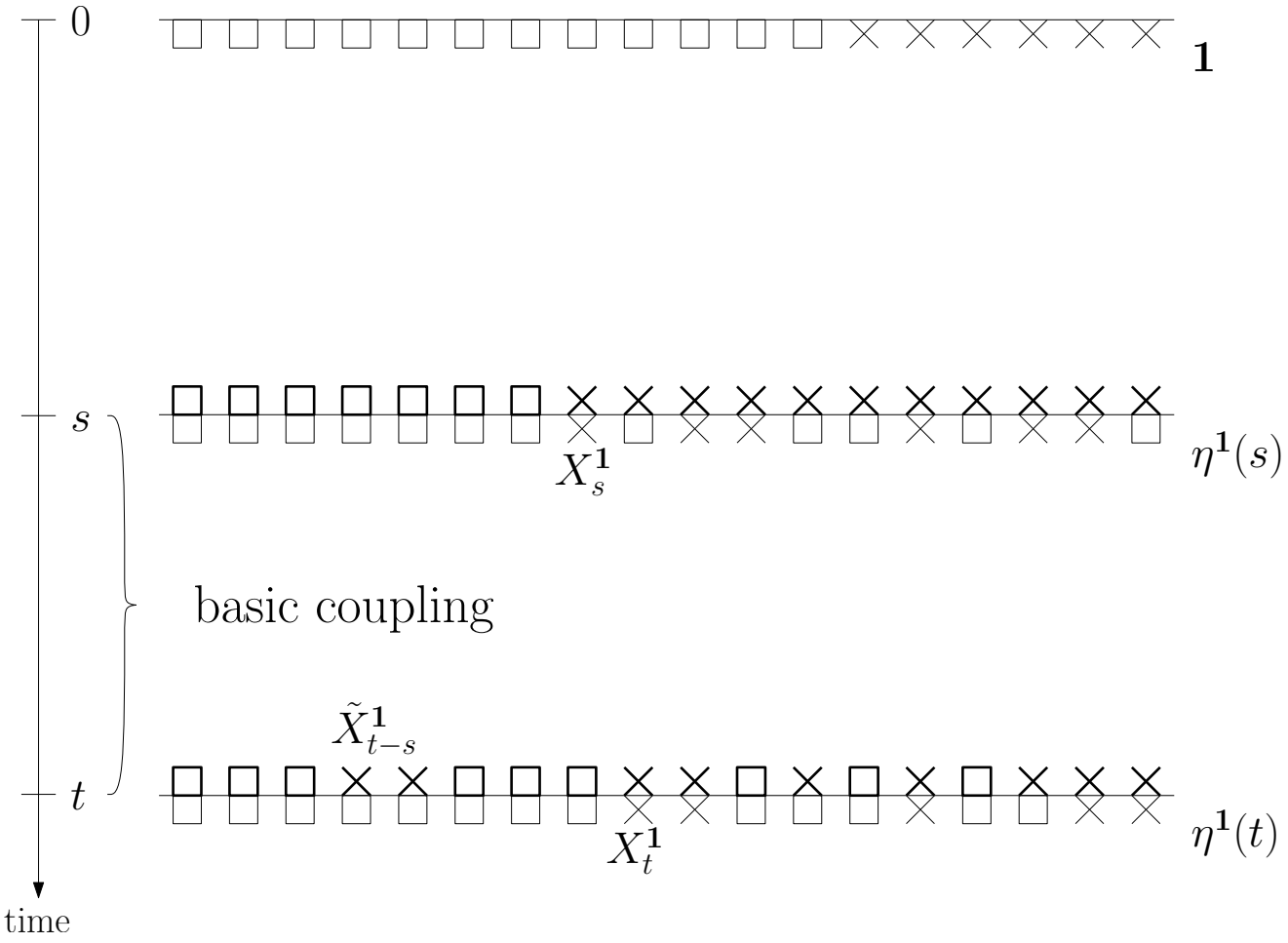
- $\frac{X_t}{t} \xrightarrow[t\to\infty]{} v < 0$?
- What does the front see? Invariant measure for $(\theta\eta(t))_{t\geq 0}$? Convergence of $(\theta\eta(t))_{t\geq 0}$?

Trouble?

No attractiveness $\Longrightarrow$ No subadditive argument.

Ex: Contact process. $\times \xrightarrow{\mathbf{1}} \square$ and $\square \xrightarrow{\lambda \cdot \sharp \sim \times} \times$.



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- *There exists $v < 0$ such that for every initial η as above*

$$\frac{X_t}{t} \underset{t\to\infty}{\longrightarrow} v \qquad \textit{in probability.}$$

- *The process seen from the front has a unique invariant measure ν and*

$$\theta\eta(t) \Longrightarrow \nu \qquad \textit{in distribution.}$$

Central argument

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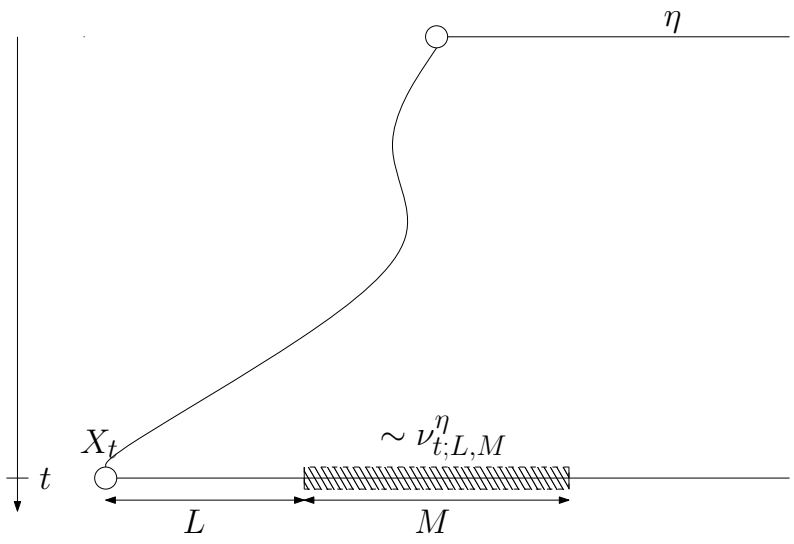

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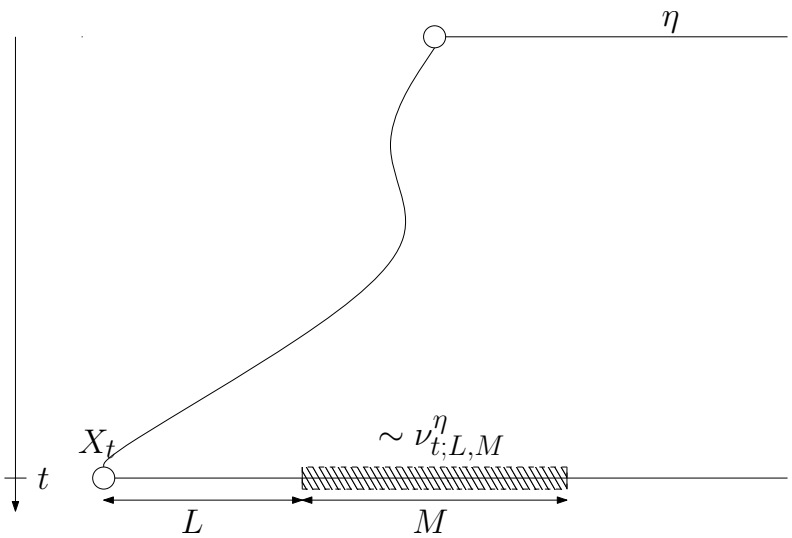

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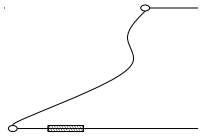
- *If* $L + M \leq Ct$ 

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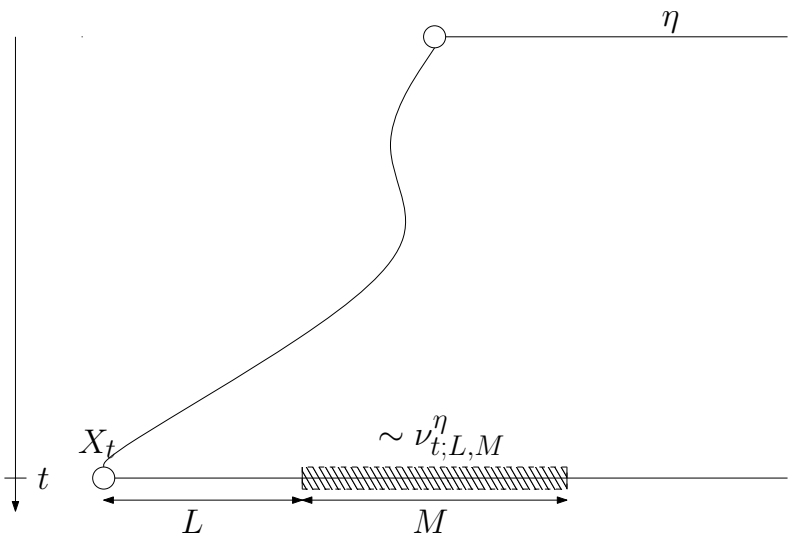

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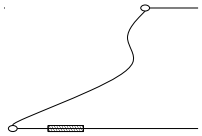
- *If* $L + M \leq Ct$ 

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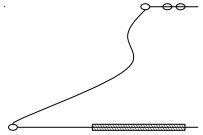
- *If* $L + M > Ct$ *and* η *has "enough zeros"* 

$$\|\nu^{\eta}_{t;L,M} - \mu\|_{TV} \leq e^{-\epsilon(L \wedge t)}$$

Thank you!