Competing islands limit the rate of adaptation in structured populations

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How fast populations do adapt?

Simplest case:

- randomly mating population of constant size N
- all mutations have the beneficial advantage s: individuals of wild type reproduce at rate 1 and individuals of beneficial type at rate $1 + s$
- beneficial mutation rate u_b is small in comparison to the selection strength, such that each beneficial mutation disappears or fixes before the next mutation occurs.

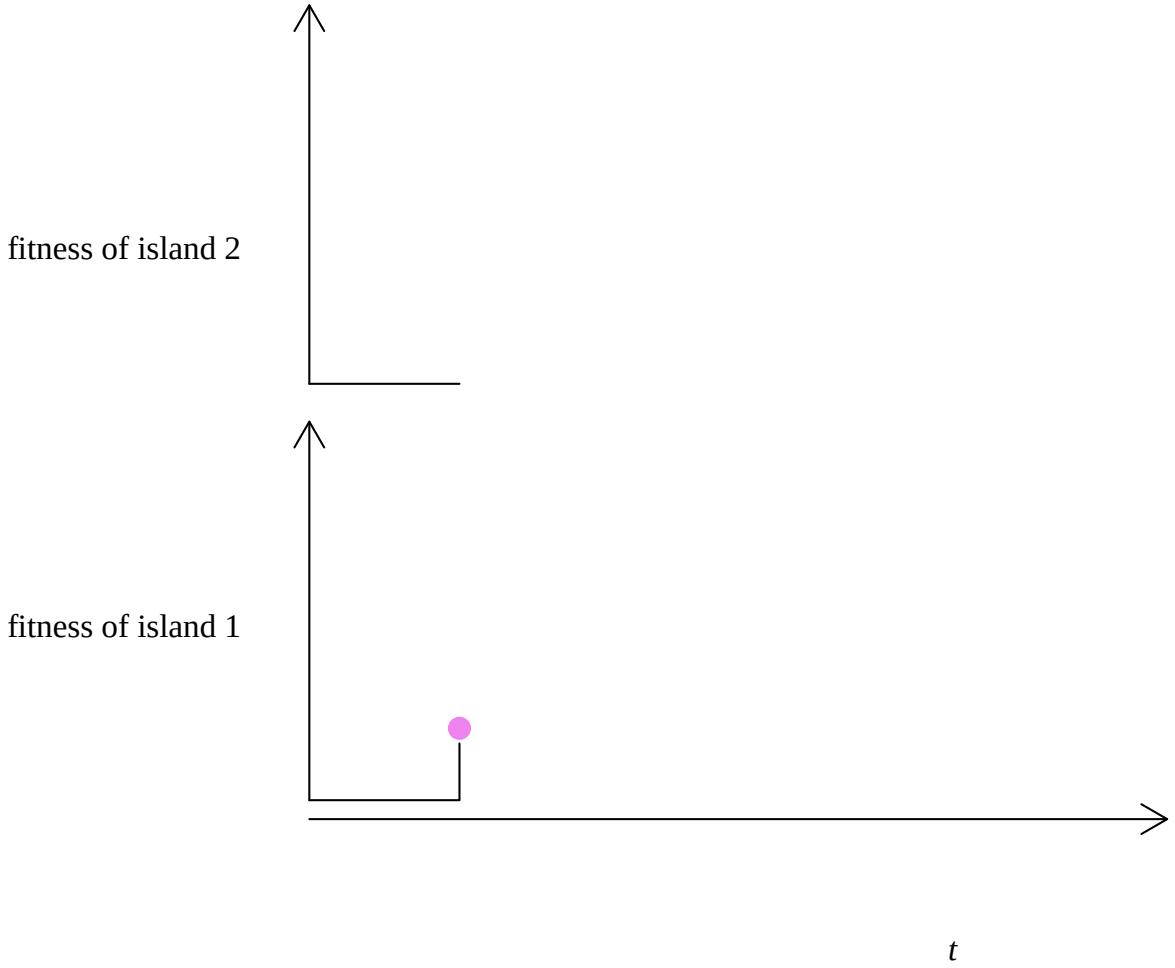
Then

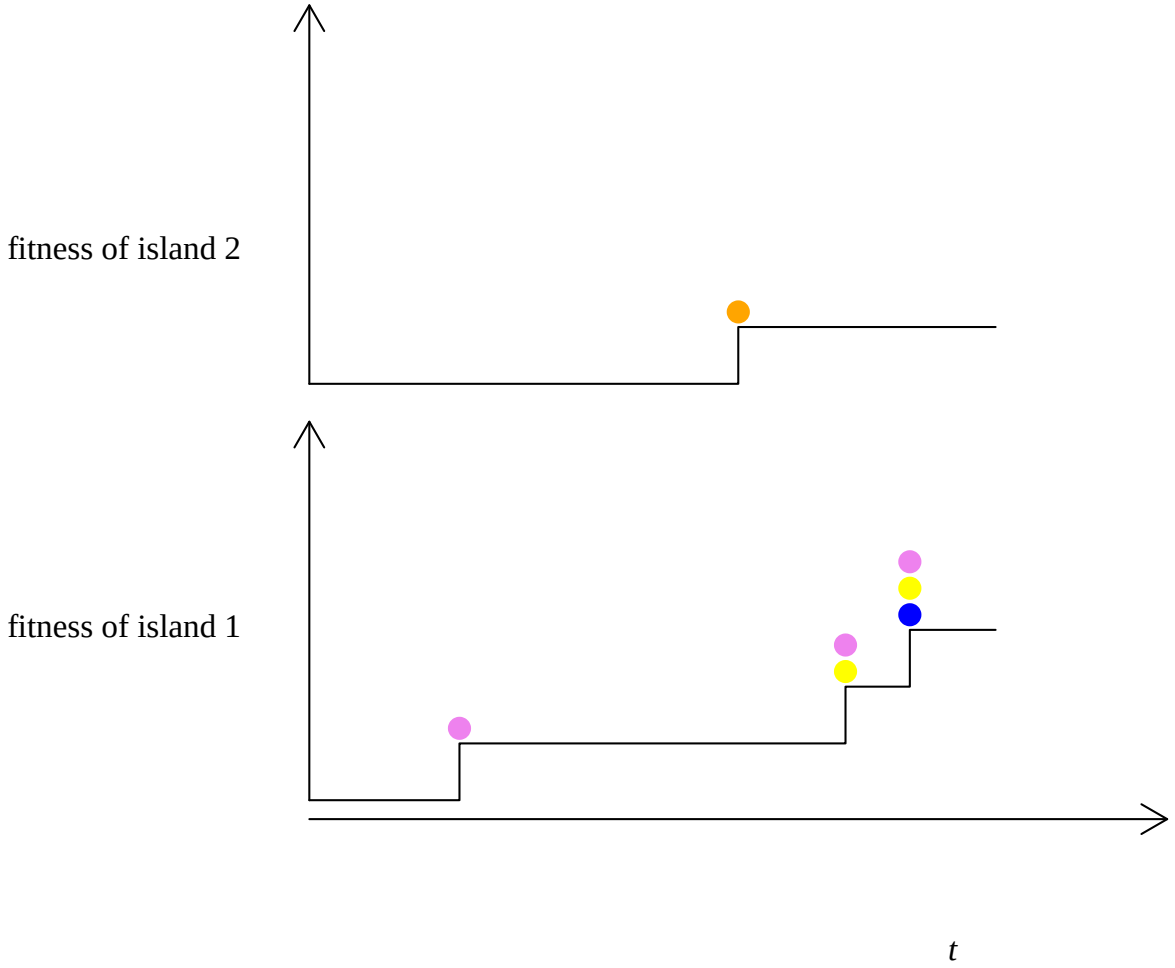
$$\text{Rate of adaptation } = s \cdot u_b \cdot N \cdot p_{\text{fix}}(s) \approx 2s^2 u_b \cdot N,$$

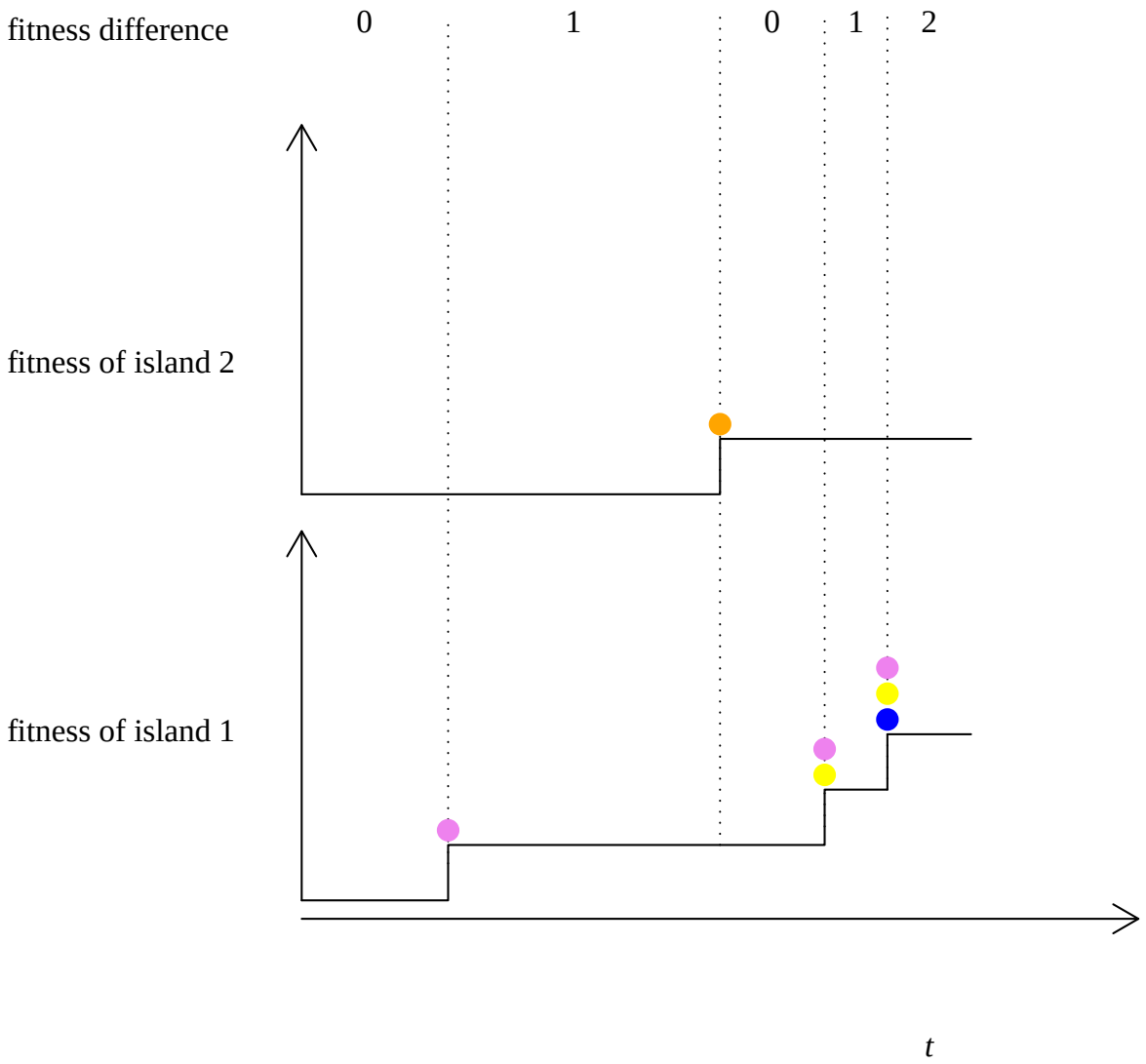
where
$p_{\text{fix}}(s) \approx 2s$ (Haldane) fixation probability of a beneficial mutation.

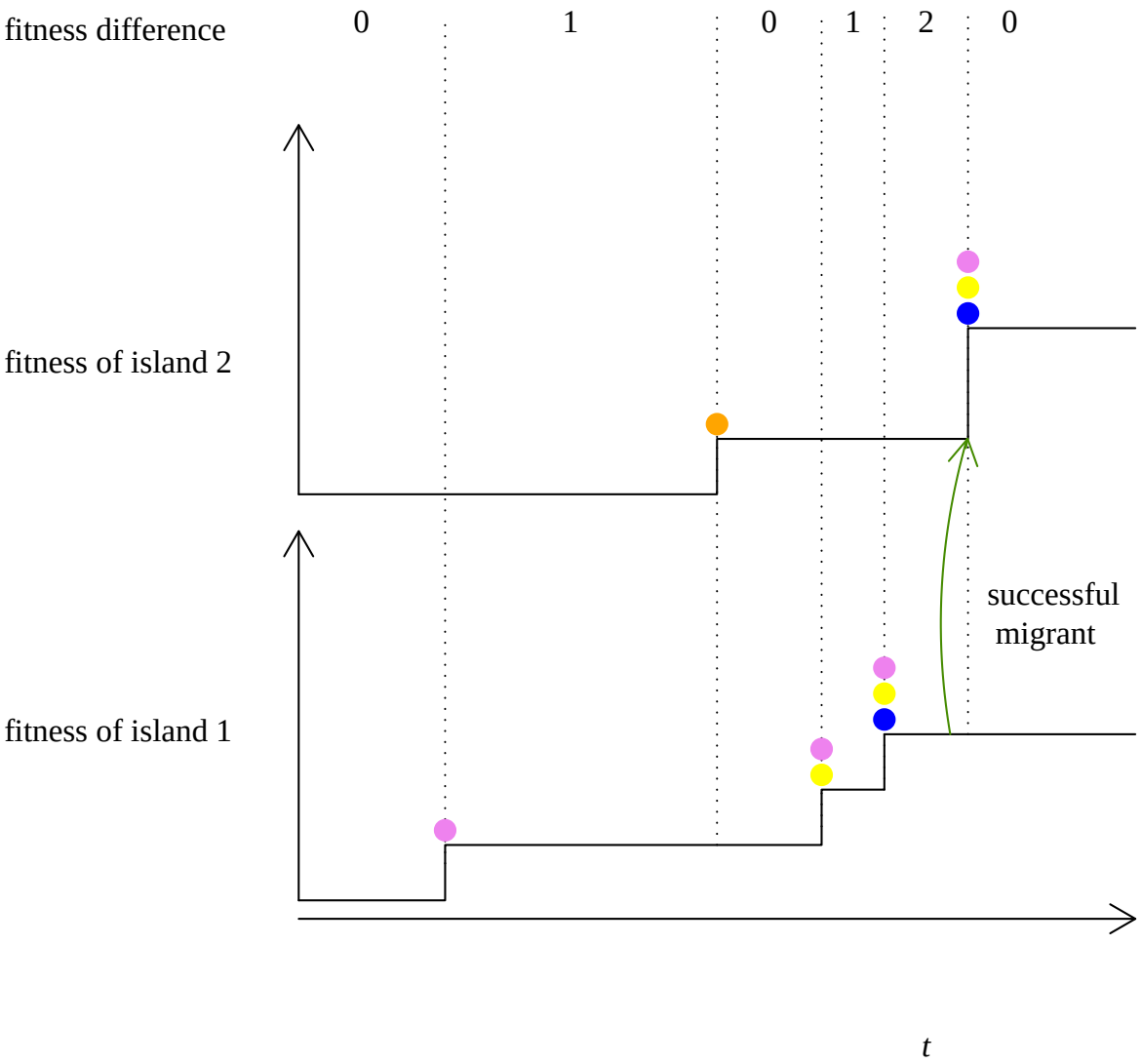
Two islands of equal size N

- random mating within island populations
- migration between islands
- only mutations with a beneficial advantage s
- beneficial mutation rate is small, such that each beneficial mutation disappears or fixes **within** an island before the next occurs.


fitness of island 2
fitness of island 1
t


fitness of island 2
fitness of island 1
t


fitness difference
0
1
0
1
2
fitness of island 2
fitness of island 1
t



After migration event:

- Mutations fixed exclusively on island 2 get lost
- All individuals have the same genotype

- If the migration rate is small, islands compete for fixation of mutations. Rate of adaptation is bounded from below by

$$s \cdot u_b \cdot N \cdot p_{\text{fix}}(s),$$

- If the migration rate is large, a mutation can even fix on the second island before the next mutation arises. No competition between islands.
  For very large migration rates

$$\text{Rate of adaptation } = s \cdot u_b \cdot 2 \cdot N \cdot p_{\text{fix}}(s).$$

How fast does the rate of adaptation decrease with the migration rate?

Successional mutations model on two islands

The SM model on two islands of equal size N and symmetric migration rate m is a jump process $(\mathcal{Z}_t)_{t\geq 0}$ with $\mathcal{Z}_t = (Z_t^1, Z_t^2)$, where

- Z_t^1 (Z_t^2) is the number of beneficial mutations fixed on island 1 (island 2).

Start $\mathcal{Z}_0 = (0,0)$,
$(Z_t^1, Z_t^2) = (z_1, z_2)$ jumps to

$$\begin{aligned}
&(z_1+1, z_2) && \text{at rate } p_{\text{fix}}(s)\cdot u_b \cdot N \\
&(z_1, z_2+1) && \text{at rate } p_{\text{fix}}(s)\cdot u_b \cdot N \\
&(\max\{z_1,z_2\}, \max\{z_1,z_2\}) && \text{at rate } p_{\text{fix}}(s\cdot|z_1-z_2|)\cdot N\cdot m,
\end{aligned}$$

where

$$p_{\text{fix}}(s\cdot|z_1-z_2|) = 2|z_1-z_2|\cdot s.$$

Theorem

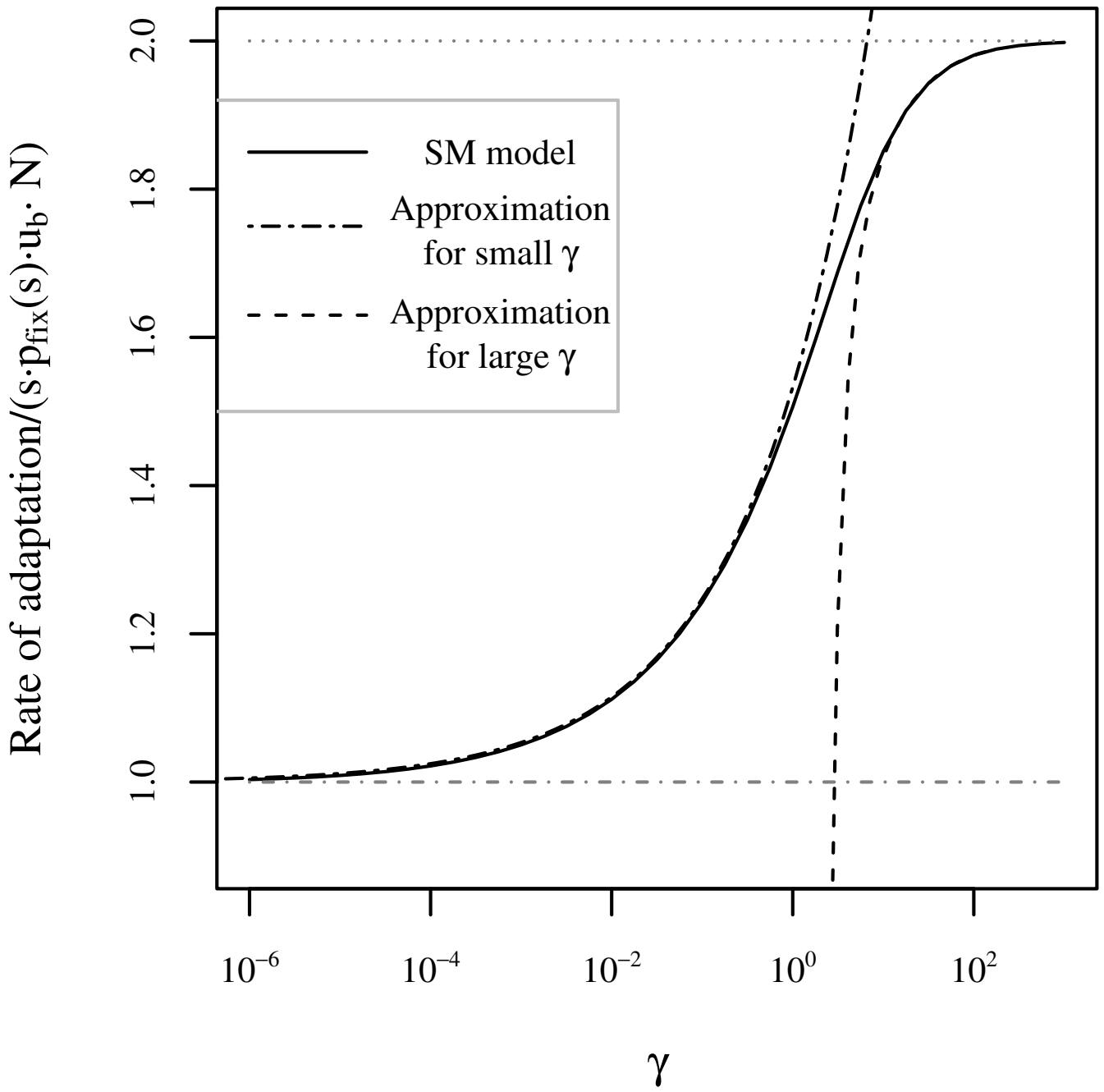
Denote by R^{SM} the rate of adaptation in SM model and $\gamma = m/u_b$. Then:

$$\lim_{m \to 0} \frac{R^{SM} - 2s^2 \cdot u_b \cdot N}{\gamma^{1/3}} = C,$$

where $C := \frac{3^{1/3}}{\Gamma(2/3)} s^2 u_b N$.

That means for small migration rates

$$R^{SM} \approx 2s^2 u_b N \cdot \left(1 + \gamma^{1/3} \cdot C\right).$$


SM model
Approximation for small γ
Approximation for large γ
Rate of adaptation/(s·p_fix(s)·u_b· N)
2.0
1.8
1.6
1.4
1.2
1.0
10^−6
10^−4
10^−2
10^0
10^2
γ

Thank you for your attention!

Read more:

Cornelia Pokalyuk, Lisha Mathew, Dirk Metzler and Peter Pfaffelhuber
Competing islands limit the rate of adaptation,
Theoretical Population Biology, in press